Question1

Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :
[27-Jan-2024 Shift 1]

Options:

A. $\log_{\frac{2}{3}} 2$

B. $\log_4 3$

C. $\log_3 4$

D. $\log_{\frac{4}{3}} 2$

Answer: D

Solution:

$$\frac{dx}{dt} + ax = 0$$

$$\frac{dx}{x} = -a\,\mathrm{dt}$$

$$\int \frac{dx}{x} = -a\int \mathrm{dt}$$

$$\ln|x| = -at + c$$

$$att = 0, x = 2$$

$$\ln 2 = 0 + c$$

$$\ln x = -at + \ln 2$$

$$\frac{x}{2} = e^{-at}$$

$$x = 2e^{-at} \ldots\ldots\ldots\ldots(\mathrm{I})$$

$$\frac{dy}{dt} + by = 0$$

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$$\frac{dy}{y} = -b\,dt$$

$$\ln|y| = -bt + \lambda$$

$t = 0, y = 1$

$0 = 0 + \lambda$

$y = e^{-bt}$ ....(II)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2e^{-a})$

$e^{a-b} = \frac{4}{3}$

For $x(t) = y(t)$

$\Rightarrow 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$2 = \left(\frac{4}{3}\right)^t$

$\log_{\frac{4}{3}} 2 = t$

Question2

If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3\log_e |2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to____ [27-Jan-2024 Shift 1]

Answer: 29

Solution:

$2x + 3y - 2 = t$

$4x + 6y - 4 = 2t$

$2 + 3\frac{dy}{dx} = \frac{dt}{dx}$

$4x + 6y - 7 = 2t - 3$

$\frac{dy}{dx} = \frac{-(2x + 3y - 2)}{4x + 6y - 7}$

$\frac{dt}{dx} = \frac{-3t + 4t - 6}{2t - 3} = \frac{t - 6}{2t - 3}$

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$$\int \frac{2t-3}{t-6}\,dt = \int dx$$

$$\int \left(\frac{2t-12}{t-6} + \frac{9}{t-6} \right) \cdot dt = x$$

$$2t + 9\ln(t-6) = x + c$$

$$2(2x+3y-2) + 9\ln(2x+3y-8) = x + c$$

$$x = 0, y = 3$$

$$c = 14$$

$$4x + 6y - 4 + 9\ln(2x+3y-8) = x + 14$$

$$x + 2y + 3\ln(2x+3y-8) = 6$$

$$\alpha = 1, \beta = 2, \gamma = 8$$

$$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$$

Question3

If y = y(x) is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, $x > 2$, $y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :
[27-Jan-2024 Shift 2]

Options:

A. $\frac{3}{1+(8)^{1/4}}$

B. $\frac{3}{1+2\sqrt{2}}$

C. $\frac{3}{1-2\sqrt{2}}$

D. $\frac{3}{1-(8)^{1/4}}$

Answer: A

Solution:

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$(x^2-4)dy-(y^2-3y)\,dx=0$

$\Rightarrow \int \frac{dy}{y^2-3y}=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}\int \frac{y-(y-3)}{y(y-3)}dy=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

At $x=4,\ y=\frac{3}{2}$

$\therefore C=\frac{1}{4}\ln 3$

$\therefore\ \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ln(3)$

At $x=10$

$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{2}{3}\right|+\frac{1}{4}\ln(3)$

$\ln\left|\frac{y-3}{y}\right|=\ln 2^{3/4},\ \forall x>2,\ \frac{dy}{dx}<0$

as $y(4)=\frac{3}{2}\Rightarrow y\in(0,3)$

$-y+3=8^{1/4}\cdot y$

$y=\frac{3}{1+8^{1/4}}$

Question4

If the solution curve, of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passing through the point (2, 1) is
$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{\beta}\log_e\left(\alpha+\left(\frac{y-1}{x-1}\right)^2\right)=\left|\log_e x-1\right|$
then $5\beta+\alpha$ is equal to__
[27-Jan-2024 Shift 2]

Answer: 11

Solution:

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$$\frac{dy}{dx} = \frac{x+y-2}{x-y}$$

$x = X + h,\ y = Y + k$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$\left.\begin{matrix} h+k-2=0 \\ h-k=0 \end{matrix}\right\} h = k = 1$$

$Y = vX$

$$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2} dv = \frac{dX}{X}$$

$$\tan^{-1} v - \frac{1}{2} \ln(1+v^2) = \ln|X| + C$$

As curve is passing through (2, 1)

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$$

$\therefore \alpha = 1$ and $\beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

Question5

A function $y = f(x)$ satisfies $f(x)\sin 2x + \sin x - (1 + \cos^2 x) f'(x) = 0$ with condition $f(0) = 0$. Then $f\left(\frac{\pi}{2}\right)$ is equal to
[29-Jan-2024 Shift 1]

Options:

A. 1

B. 0

C. -1

D. 2

Answer: A

Solution:

Solution:

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$$\frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\sin x$$

$$\text{I.F.} = 1+\cos^2 x$$

$$y\cdot(1+\cos^2 x)=\int(\sin x)\,dx$$

$$=-\cos x+C$$

$$x=0,\ C=1$$

$$y\left(\frac{\pi}{2}\right)=1$$

Question6

If the solution curve y = y(x) of the differential equation $(1+y^2)(1+\log_e x)\,dx + x\,dy = 0$, x > 0 passes through the point (1, 1) and $y(e)=\dfrac{\alpha-\tan\left(\frac{3}{2}\right)}{\beta+\tan\left(\frac{3}{2}\right)}$, then α + 2β is

[29-Jan-2024 Shift 1]

Answer: 3

Solution:

$$\int\left(\frac{1}{x}+\frac{\ln x}{x}\right)dx+\int\frac{dy}{1+y^2}=0$$

$$\ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=C$$

Put $x=y=1$

$$\therefore C=\frac{\pi}{4}$$

$$\Rightarrow \ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=\frac{\pi}{4}$$

Put $x=e$

$$\Rightarrow y=\tan\left(\frac{\pi}{4}-\frac{3}{2}\right)=\frac{1-\tan\frac{3}{2}}{1+\tan\frac{3}{2}}$$

$$\therefore \alpha=1,\ \beta=1$$

$$\Rightarrow \alpha+2\beta=3$$

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Question7

If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y = \frac{\pi}{3}$, then α^2 is equal to
[29-Jan-2024 Shift 2]

Options:

A. 3

B. 12

C. 4

D. 9

Answer: A

Solution:

Solution:

Differential equation :-

$$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$$

$$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$$

Divide both sides by x^2

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$$

Let $\frac{y}{x} = t$

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$$

$$\cos tt = \frac{1}{x}dx$$

Integrating both sides

$$\sin t = \ln|x| + c$$

$$\sin\frac{y}{x} = \ln|x| + c$$

Using $y(1) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

Question8

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$(1-x^2)dy = [xy + (x^3+2)\sqrt{3(1-x^2)}]\,dx$ $-1 < x < 1$, $y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are coprime numbers, then m + n is equal to
[30-Jan-2024 Shift 1]

Answer: 97

Solution:

Solution:

$$\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$$

$$\text{IF} = e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$$

$$y\sqrt{1-x^2} = \sqrt{3}\int(x^3+2)\,dx$$

$$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4}+2x\right)+c$$

$$\Rightarrow y(0) = 0 \quad \therefore c = 0$$

$$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$$

$$m+n = 97$$

Question9

Let $y = y(x)$ be the solution of the differential equation $\sec x\,dy + \{2(1-x)\tan x + x(2-x)\}\,dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to :
[30-Jan-2024 Shift 1]

Options:

A. 2

B. $2\{1-\sin(2)\}$

C. $2\{\sin(2)+1\}$

D. 1

Answer: A

Solution:

Solution:

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$$\frac{dy}{dx} = 2(x-1)\sin x + (x^2 - 2x)\cos x$$

Now both side integrate

$$y(x) = \int 2(x-1)\sin x\, dx + [(x^2-2x)(\sin x) - \int (2x-2)\sin x\, dx]$$

$$y(x) = (x^2 - 2x)\sin x + \lambda$$

$$y(0) = 0 + \lambda \Rightarrow 2 = \lambda$$

$$y(x) = (x^2 - 2x)\sin x + 2$$

$$y(2) = 2$$

Question10

The solution curve of the differential equation $y\frac{dx}{dy} = x(\log_e x - \log_e y + 1), x > 0, y > 0$ passing through the point (e, 1) is
[31-Jan-2024 Shift 1]

Options:

A. $\left|\log_e \frac{y}{x}\right| = x$

B. $\left|\log_e \frac{y}{x}\right| = y^2$

C. $\left|\log_e \frac{x}{y}\right| = y$

D. $2\left|\log_e \frac{x}{y}\right| = y + 1$

Answer: C

Solution:

$$\frac{dx}{dy} = \frac{x}{y}\left(\ln\left(\frac{x}{y}\right) + 1\right)$$

Let $\frac{x}{y} = t \Rightarrow x = ty$

$$\frac{dx}{dy} = t + y\frac{dt}{dy}$$

$$t + y\frac{dt}{dy} = t(\ln(t) + 1)$$

$$y\frac{dt}{dy} = t\ln(t) \Rightarrow \frac{dt}{t\ln(t)} = \frac{dy}{y}$$

$$\Rightarrow \int \frac{dt}{t\cdot\ln(t)} = \int \frac{dy}{y}$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{dy}{y}$$

let $\ln t = p$

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$\frac{1}{t}\,dt = dp$

$\Rightarrow \ln p = \ln y + c$

$\ln(\ln t) = \ln y + c$

$\ln\left(\ln\left(\frac{x}{y}\right)\right) = \ln y + c$

at $x = e, y = 1$

$\ln\left(\ln\left(\frac{e}{1}\right)\right) = \ln(1) + c \Rightarrow c = 0$

$\ln\left|\ln\left(\frac{x}{y}\right)\right| = \ln y$

$\left|\ln\left(\frac{x}{y}\right)\right| = e^{\ln y}$

$\left|\ln\left(\frac{x}{y}\right)\right| = y$

Question11

Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then, $y\left(\frac{\pi}{3}\right)$ is

[31-Jan-2024 Shift 1]

Options:

A. $\sqrt{3}(2 + \log_e \sqrt{3})$

B. $\frac{\sqrt{3}}{2}(2 + \log_e 3)$

C. $\sqrt{3}(1 + 2\log_e 3)$

D. $\sqrt{3}(2 + \log_e 3)$

Answer: A

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Solution:

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x \cdot \cos x\left(\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x}\right)}$$

$$= \frac{\sin x + y\cos x}{\sin x(1 - \sin^2 x)}$$

$$\frac{dy}{dx} = \sec^2 x + y \cdot 2(\operatorname{cosec} 2x)$$

$$\frac{dy}{dx} - 2\operatorname{cosec}(2x) \cdot y = \sec^2 x$$

$$\frac{dy}{dx} + p \cdot y = Q$$

$$\text{I.F.} = e^{\int p dx} = e^{\int -2\operatorname{cosec}(2x)\, dx}$$

$$\text{Let } 2x = t$$

$$2\frac{dx}{dt} = 1$$

$$dx = \frac{dt}{2}$$

$$= e^{-\int \operatorname{cosec}(t)\, dt}$$

$$= e^{-\ln\left|\tan\frac{t}{2}\right|}$$

$$= e^{-\ln|\tan x|} = \frac{1}{|\tan x|}$$

$$y(\text{IF}) = \int Q(\text{IF})\, dx + c$$

$$\Rightarrow y\frac{1}{|\tan x|} = \int \sec^2 x \cdot \frac{1}{|\tan x|} + c$$

$$y \cdot \frac{1}{|\tan x|} = \int \frac{dt}{|t|} + c \quad \text{for } \tan x = t$$

$$y \cdot \frac{1}{|\tan x|} = \ln|t| + c$$

$$y = |\tan x|(\ln|\tan x| + c)$$

$$\text{Put } x = \frac{\pi}{4}, y = 2$$

$$2 = \ln 1 + c \Rightarrow c = 2$$

$$y = |\tan x|(\ln|\tan x| + 2)$$

$$y\left(\frac{\pi}{3}\right) = \sqrt{3}(\ln\sqrt{3} + 2)$$

Question12

The temperature T(t) of a body at time t = 0 is 160° F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T-80)$, where K

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[31-Jan-2024 Shift 2]

Options:

A. 85°F

B. 95°F

C. 90°F

D. 80°F

Answer: C

Solution:

Solution:

$$\frac{dT}{dt} = -k(T-80)$$

$$\int_{160}^{T} \frac{dT}{(T-80)} = \int_{0}^{t} -K\,dt$$

$$[\ln|T-80|]_{160}^{T} = -kt$$

$$\ln|T-80| - \ln 80 = -kt$$

$$\ln\left|\frac{T-80}{80}\right| = -kt$$

$$T = 80 + 80e^{-kt}$$

$$120 = 80 + 80e^{-k \cdot 15}$$

$$\frac{40}{80} = e^{-k15} = \frac{1}{2}$$

$$\therefore T(45) = 80 + 80e^{-k \cdot 45}$$

$$= 80 + 80(e^{-k.15})^3$$

$$= 80 + 80 \times \frac{1}{8}$$

$$= 90$$

Question13

Let $y = y(x)$ be the solution of the differential equation $\sec^2 x\,dx + (e^{2y}\tan^2 x + \tan x)\,dy = 0$

$0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$

Then $e^{8\alpha}$ is equal to

[31-Jan-2024 Shift 2]

Answer: 9

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Solution:

$$sec^2x\frac{dx}{dy}+e^{2y}\tan^2x+\tan x=0$$

$$\left(\text{Put } \tan x=t\Rightarrow sec^2x\frac{dx}{dy}=\frac{dt}{dy}\right)$$

$$\frac{dt}{dy}+e^{2y}\times t^2+t=0$$

$$\frac{dt}{dy}+t=-t^2\cdot e^{2y}$$

$$\frac{1}{t^2}\frac{dt}{dy}+\frac{1}{t}=-e^{2y}$$

$$\left(\text{Put } \frac{1}{t}=u\ \frac{-1}{t^2}\frac{dt}{dy}=\frac{du}{dy}\right)$$

$$\frac{-du}{dy}+u=-e^{2y}$$

$$\frac{du}{dy}-u=e^{2y}$$

$$\text{I.F.}\ =e^{-\int dy}=e^{-y}$$

$$ue^{-y}=\int e^{-y}\times e^{2y}dy$$

$$\frac{1}{\tan x}\times e^{-y}=e^{y}+c$$

$$x=\frac{\pi}{4},y=0,c=0$$

$$x=\frac{\pi}{6},y=\alpha$$

$$\sqrt{3}e^{-\alpha}=e^{\alpha}+0$$

$$e^{2\alpha}=\sqrt{3}$$

$$e^{8\alpha}=9$$

Question14

Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=2x(x+y)^3-x(x+y)-1$, $y(0)=1$.

Then, $\left(\frac{1}{\sqrt{2}}+y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

[1-Feb-2024 Shift 1]

Options:

A. $\frac{4}{4+\sqrt{e}}$

B. $\frac{3}{3-\sqrt{e}}$

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D. $\frac{1}{2-\sqrt{e}}$

Answer: D

Solution:

$$\frac{dy}{dx}=2x(x+y)^3-x(x+y)-1$$

$$x+y=t$$

$$\frac{dt}{dx}-1=2xt^3-xt-1$$

$$\frac{dt}{2t^3-t}=x\,dx$$

$$\frac{tdt}{2t^4-t^2}=xdx$$

Let $t^2=z$

$$\int\frac{dz}{2(2z^2-z)}=\int x\,dx$$

$$\int\frac{dz}{4z\left(z-\frac{1}{2}\right)}=\int x\,dx$$

$$\ln\left|\frac{z-\frac{1}{2}}{z}\right|=x^2+k$$

$$z=\frac{1}{2-\sqrt{e}}$$

Question15

If x = x(t) is the solution of the differential equation $(t+1)\,dx=(2x+(t+1)^4)\,dt$, x(0) = 2, then, x(1) equals ________ [1-Feb-2024 Shift 1]

Answer: 14

Solution:

$(t+1)\,dx = (2x+(t+1)^4)\,dt$

$\frac{dx}{dt} = \frac{2x+(t+1)^4}{t+1}$

$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$

$I \cdot F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$

$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3\,dt + c$

$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$

$\Rightarrow c = \frac{3}{2}$

$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$

put, $t = 1$

$x = 2^3 + 6 = 14$

Question16

Let α be a non-zero real number. Suppose $f : R \longrightarrow R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \longrightarrow -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to
[1-Feb-2024 Shift 2]

Options:

A. 3

B. 5

C. 9

D. 7

Answer: A

Solution:

Solution:
$f(0) = 2,\ \lim_{x \to -\infty} f(x) = 1$
$f'(x) - \alpha \cdot f(x) = 3$
$I \cdot F = e^{-\alpha x}$
$y(e^{-\alpha x}) = \int 3 \cdot e^{-\alpha x}\,dx$
$f(x) \cdot (e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha} + c$
$x = 0 \Rightarrow 2 = \frac{-3}{\alpha} + c \Rightarrow \frac{3}{\alpha} = c - 2$
$f(x) = \frac{-3}{\ } + c \cdot e^{\alpha x}$

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$x \to -\infty \Rightarrow 1 = \frac{-3}{\alpha} + c(0)$

$\alpha = -3 \therefore c = 1$

$f(-\ln 2) = \frac{-3}{\alpha} + c \cdot e^{\alpha x}$

$= 1 + e^{3 \ln 2} = 9$

(But α should be greater than 0 for finite value of c)

Question17

If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, x(1) = 1, then 5x(2) is equal to :
[1-Feb-2024 Shift 2]

Answer: 5

Solution:

Solution:

$\frac{dx}{dy} - \frac{x}{y} = \frac{1-y^2}{y}$

Integrating factor $= e^{\int -\frac{1}{y}} = \frac{1}{y}$

$x \cdot \frac{1}{y} = \int \frac{1-y^2}{y^2} dy$

$\frac{x}{y} = \frac{-1}{y} - y + c$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

$5x(2) = 5(-1 - 4 + 6)$

$= 5$

Question18

Let y = y(x) be the solution of the differential equation
$x^3 dy + (xy - 1) dx = 0$, $x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then y(1) is equal to
[24-Jan-2023 Shift 1]

Options:

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B. e

C. $2 - e$

D. 3

Answer: A

Solution:

Solution:

$$\frac{dy}{dx} = \frac{1 - xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}$$

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$$

$$\text{If } = e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$$

$$y \cdot e^{-\frac{1}{x}} = \int e^{-\frac{1}{x}} \cdot \frac{1}{x^3} dx \left(\text{put } -\frac{1}{x} = t \right)$$

$$y \cdot e^{-\frac{1}{x}} = -\int e^t \cdot t\,dt$$

$$y = \frac{1}{x} + 1 + Ce^{\frac{1}{x}}$$

Where C is constant

Put $x = \frac{1}{2}$

$$3 - e = 2 + 1 + Ce^2$$

$$C = -\frac{1}{e}$$

$$y(1) = 1$$

Question19

Let $y = y(x)$ be the solution of the differential equation left$(x^2 - 3y^2$ right$)\,dx + 3xy\,dy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to [24-Jan-2023 Shift 2]

Options:

A. $3e^2$

B. e^2

C. $2e^2$

D. $\frac{3e^2}{2}$

Answer: C

Solution:

Solution:

$$(x^2 - 3y^2)\,dx + 3xy\,dy = 0$$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{\ldots} \Rightarrow \frac{dy}{dx} = \frac{y}{\ldots} - \frac{1}{\ldots}\frac{x}{\ldots}$$

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$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$

$\Rightarrow v\,dv = \frac{-1}{3x}$

Integrating both side

$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$

$y(1) = 1$

$\Rightarrow \frac{1}{2} = c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$

$\Rightarrow y^2 = -\frac{2}{3}x^2\ln x + x^2$

$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow 6y^2(e) = 2e^2$

Question20

Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}(1 + xy^2(1 + \log_e x)), x > 0, y(1) = 3$. Then $\frac{y^2(x)}{9}$ is equal to : [25-Jan-2023 Shift 1]

Options:

A. $\frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$

B. $\frac{x^2}{2x^3(2 + \log_e x^3) - 3}$

C. $\frac{x^2}{3x^3(1 + \log_e x^2) - 2}$

D. $\frac{x^2}{7 - 3x^3(2 + \log_e x^2)}$

Answer: A

Solution:

Solution:

$\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \log_e x)$

$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{xyy^2} = 1 + \log_e x$

Let $-\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} + \frac{2t}{x} = 2(1 + \log_e x)$

I.F. $= e^{\int \frac{2}{x}dx} = x^2$

$-x^2 \quad 2\left((1 + \log_e x)x^3 \quad x^3 \right) + C$

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$$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$$

$$x\,dy = y\,dx + xy^3(1 + \log_e x)\,dx$$

$$\frac{x\,dy - y\,dx}{y^3} = x(1 + \log_e x)\,dx$$

$$-\frac{x}{y} d\left(\frac{x}{y}\right) = x^2(1 + \log_e x)\,dx$$

$$-\left(\frac{x}{y}\right)^2 = 2\int x^2(1 + \log_e x)\,dx$$

Question21

Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$
Where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim_{t \to \infty} y(t)$

[25-Jan-2023 Shift 2]

Options:

A. is 0

B. does not exist

C. is 1

D. is - 1

Answer: A

Solution:

Solution:

$$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$$

$$\text{I.F.} = e^{\int \alpha\,dt} = e^{\alpha t}$$

$$\text{Solution} \Rightarrow y \cdot e^{\alpha t} = \int \gamma e^{-\beta T} \cdot e^{\alpha t}\,dt$$

$$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha - \beta)t}}{(\alpha - \beta)} + c$$

$$\Rightarrow y = \frac{\gamma}{e^{\beta t}(\alpha - \beta)} + \frac{c}{e^{\alpha t}}$$

$$\text{So, } \lim_{t \to \infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$$

Question22

Let $y = f(x)$ be the solution of the differential equation $y(x + 1)\,dx - x^2\,dy = 0$, $y(1) = e$. Then $\lim_{x \to 0^+} f(x)$ is equal to

[29-Jan-2023 Shift 1]

Options:

A. 0

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B. $\frac{1}{e}$

C. e^2

D. $\frac{1}{e^2}$

Answer: A

Solution:

Solution:

$\frac{x+1}{x^2}dx = \frac{dy}{y}$

$\ln x - \frac{1}{x} = \ln y + c$

$(1, e)$

$c = -2$

$\ln x - \frac{1}{x} = \ln y - 2$

$y = e^{\ln x} - \frac{1}{x} + 2$

$\lim_{x \to 0^+} e^{\ln x - 1} - \frac{1}{x} + 2$

$= e^{-\infty}$

$= 0$

Question23

Let $y = y(x)$ be the solution of the differential equation $x\log_e x \frac{dy}{dx} + y = x^2\log_e x$, $(x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to [29-Jan-2023 Shift 2]

Options:

A. $\frac{4+e^2}{4}$

B. $\frac{1+e^2}{4}$

C. $\frac{2+e^2}{2}$

D. $\frac{1+e^2}{2}$

Answer: A

Solution:

Solution:

$x\log_e x \frac{dy}{dx} + y = x^2\log_e x, (x > 1)$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\ln x} = x$

Linear differential equation

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∴ Solution of differential equation

$y\,|\ln x| = \int x\,|\ln x|\,dx$

$= |\ln x| \,\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx$

$\Rightarrow y\,|\ln x| = |\ln x|\left(\frac{x^2}{2}\right) - \frac{x^2}{4} + c$

For constant

$y(2) = 2 \Rightarrow c = 1$

So, $y(x) = \frac{x^2}{2} - \frac{x^2}{4\,|\ln x|} + \frac{1}{|\ln x|}$

Hence, $y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = 1 + \frac{e^2}{4}$

Question24

Let the solution curve y = y(x) of the differential equation $\frac{dy}{dx} - \frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}}y = 2x$ $\exp\frac{x^3 - \tan^{-1}x^3}{\sqrt{(1+x)^6}}$ pass through the origin. Then y(1) is equal to : [30-Jan-2023 Shift 1]

Options:

A. $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$

B. $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

C. $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$

D. $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

Answer: A

Solution:

$$\frac{dy}{dx} + \left(\frac{-3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}\right)y = 2e^{\left\{\frac{x-\tan x}{\sqrt{1+x^6}}\right\}}$$

$$\text{I.F.} = e^{\int\frac{-3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$$

$$= e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}}$$

Solution of differential equation

$$y\cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = \int 2xe^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}\cdot e^{\left(\frac{\tan^{-1}(x^3)-x^3}{\sqrt{1+x^6}}\right)}dx$$

$$= \int 2x\,dx + c$$

$$y\cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = x^2 + c$$

Also it passes through origin

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$c = 0$

$$y(1) \cdot e^{\frac{\tan^{-1}(1) - 1}{\sqrt{2}}} = 1$$

$$y(1) \cdot e^{\frac{\frac{\pi}{4} - 1}{\sqrt{2}}} = 1$$

$$y(1) \cdot e^{\frac{\pi - 4}{4\sqrt{2}}} = 1$$

$$y(1) = \frac{1}{e^{\frac{\pi - 4}{4\sqrt{2}}}} = e^{\frac{4 - \pi}{4\sqrt{2}}}$$

Question25

The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2 + 3y^2}{3x^2 + y^2}\right)$, $y(1) = 0$ is

[30-Jan-2023 Shift 2]

Options:

A. $\log_e |x + y| - \frac{xy}{(x + y)^2} = 0$

B. $\log_e |x + y| + \frac{xy}{(x + y)^2} = 0$

C. $\log_e |x + y| + \frac{2xy}{(x + y)^2} = 0$

D. $\log_e |x + y| - \frac{2xy}{(x + y)^2} = 0$

Answer: C

Solution:

Solution:

Put $y = vx$

$$v + x\frac{dv}{dx} = -\left(\frac{1 + 3v^2}{3 + v^2}\right)$$

$$x\frac{dv}{dx} = -\frac{(v + 1)^3}{3 + v^2}$$

$$\frac{(3 + v^2)dv}{(v + 1)^3} + \frac{dx}{x} = 0$$

$$\int \frac{4dv}{(v + 1)^3} + \int \frac{dv}{v + 1} - \int \frac{2dv}{(v + 1)^2} + \int \frac{dx}{x} = 0$$

$$\frac{-2}{(v + 1)^2} + \ln(v + 1) + \frac{2}{v + 1} + \ln x = c$$

$$\frac{-2x^2}{(x + y)^2} + \ln\left(\frac{x + y}{x}\right) + \frac{2x}{x + y} + \ln x = c$$

$$\frac{2xy}{(x + y)^2} + \ln(x + y) = c$$

$\therefore c = 0$, as $x = 1, y = 0$

$$\therefore \frac{2xy}{(x+y)^2} + \ln(x + y) = 0$$

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Question26

Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)dy = 0$ such that $y(1) = 1$. then $|(y(2))^3 - 12y(2)|$ is equal to:
[31-Jan-2023 Shift 2]

Options:

A. $32\sqrt{2}$

B. 64

C. $16\sqrt{2}$

D. 32

Answer: A

Solution:

Solution:

$(3y^2 - 5x^2)y \cdot dx + 2x(x^2 - y^2)dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y(5x^2 - 3y^2)}{2x(x^2 - y^2)}$

Put $y = mx$

$\Rightarrow m + x \cdot \frac{dm}{dx} = \frac{m(5 - 3m^2)}{2(1 - m^2)}$

$x \cdot \frac{dm}{dx} = \frac{(5 - 3m^2)m - 2m(1 - m^2)}{2(1 - m^2)}$

$\Rightarrow \frac{dx}{x} = \frac{2(m^2 - 1)}{m(m^2 - 3)} dm$

$\Rightarrow \frac{dx}{x} = \left(\frac{2}{m} - \frac{\frac{4}{3}}{m} + \frac{\frac{4m}{3}}{m^2 - 3} \right) dm$

$\Rightarrow \int \frac{dx}{x} = \int \frac{\left(\frac{2}{3}\right)}{m} + \int \frac{2}{3}\left(\frac{2m}{m^2 - 3} \right) dm$

$\Rightarrow \ln|x| = \frac{2}{3}\ln|m| + \frac{2}{3}\ln|m^2 - 3| + C$

Or, $\ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| + C$

Put $(x = 1, y = 1)$: we get $c = -\frac{2}{3}\ln(2)$

$\Rightarrow \ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| - \frac{2}{3}\ln(2)$

$\Rightarrow \left(\frac{y}{x}\right)\left[\left(\frac{y}{x}\right)^2 - 3\right] = 2 \cdot (x^{3/2})$

Put $x = 2$ to get $y(2)$

$\Rightarrow y(y^2 - 12) = 4 \times 2 \times 2 \times 2\sqrt{2}$

$\Rightarrow y^3 - 12y = 32\sqrt{2}$

$\Rightarrow |y^3(2) - 12y(2)| = 32\sqrt{2}$

Question27

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If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x$, $0 \le x \le \frac{\pi}{3}$, $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to
[1-Feb-2023 Shift 1]

Options:

A. $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

B. $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

C. $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

D. $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Answer: A

Solution:

Solution:
Here I.F. $=$ secx
Then solution of D.E :
$y(\sec x) = x\tan x - \ln(\sec x) + c$
Given $y(0) = 1 \Rightarrow c = 1$
$\therefore y(\sec x) = x\tan x - \ln(\sec x) + 1$
At $x = \frac{\pi}{6}, y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

Question28

Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2yd\,y - (1 - xy^2)\,dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :
[1-Feb-2023 Shift 2]

Options:

A. 1

B. -1

C. 0

D. 3

Answer: A

Solution:

Solution:
$\alpha x = e^{x^8 \cdot y^y}$

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$2x^2y\frac{dy}{dx} = 1 - x \cdot y^2 \quad y^2 = t$

$x^2\frac{dt}{dx} = 1 - xt$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \quad \text{I.F.} \quad = e^{tnx} = x$

$t(x) = \int \frac{1}{x^2} \cdot x\,dx$

$y^2 \cdot x = \ell nx + C$

$\therefore 2 . \ln 2 = \ln 2 + C$

$\therefore C = \ell n2$

Hence, $xy^2 = \ell n2x$

$\therefore 2x = e^{x \cdot y^2}$

Hence $\alpha = 2, \beta = 1, \gamma = 2$

Question29

Let $y = y(x)$ be a solution of the differential $(x\cos x)dy + (xy\sin x + y\cos x - 1)\,dx = 0,\ 0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ______.

[6-Apr-2023 shift 1]

Answer: 2

Solution:

Solution:

$(x\cos x)dy + (xy\sin x + y\cos x - 1)\,dx = 0,\ 0 < x < \frac{\pi}{2}$

$\frac{dy}{dx} + \left(\frac{x\sin x + \cos x}{x\cos x}\right)y = \frac{1}{x\cos x}$

$\text{I F} = x\sec x$

$y \cdot x\sec x = \int \frac{x\sec x}{x\cos x}dx = \tan x + c$

Since $y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$ Hence $c = \sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$

Question30

If the solution curve $f(x, y) = 0$ of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y,\ x > 0$, passes through the points $(1, 0)$ and $(\alpha, 2)$, then α^α is equal to :

[6-Apr-2023 shift 2]

Options:

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A. $e^{\sqrt{2c^2}}$

B. e^{c^2}

C. $e^{2e^{\sqrt{2}}}$

D. e^{2c^2}

Answer: D

Solution:

Solution:

$(1 + \ell \ln x)\frac{dx}{dy} - x\ell \ln x = e^y$

Let $x\ell nx = t$

$(1 + \ln x)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y \quad P = -1, Q = e^y$

$I \cdot F = e^{\int -dy} = e^{-y}$

Solution -

$(t)(e^{-y}) = \int (e^{-y})(e^y)dy$

$t(e^{-y}) = y + c$

$(x\ell \ln x)e^{-y} = y + c \quad \text{pass } (1, 0) \Rightarrow c = 0$

$\text{pass } (\alpha, 2)$

$\alpha^\alpha = e^{2e^2}$

Ans. Option 4

Question31

Let the solution curve $x = x(y)$, $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x\log_e.(\cos y))\sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to
[8-Apr-2023 shift 2]

Answer: 12

Solution:

$\text{Cos } y \ln^2 \cos y\, dx = (1 + 3x \ln \cos y)\sin y\, dy$

$\frac{dx}{dy} = \tan y\left(\frac{3x}{\ln \cos y} + \frac{1}{\ln^2 \cos y}\right)$

$\frac{dx}{dy} - \left(\frac{3\tan y}{\ln \cos y}\right)x = \frac{\tan y}{\ln^2 \cos y}$

$\text{If } = e^{-\frac{3}{\ln \cos y \cos y}dy}$

$\ln \cos y = t$

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$\frac{1}{\cos y} - \sin y d y = d t$

If $= e^{\frac{3}{t} dt} = e^{3 \ln t} = t^3 = \ln^3 \cos y$

solution is $x \cdot \ln^3 \cos y = \int \frac{\sin y}{\cos y} \cdot \ln \cos y d y + C.$

$x \ln^3 \cos y = \frac{-\ln^2 \cos y}{2} + C$

$x\left(\frac{\pi}{3}\right) = \frac{1}{2 \ln 2}$ so $\frac{1}{2 \ln^2} \times \ln^3\left(\frac{1}{2}\right) = -\frac{\ln^3\left(\frac{1}{2}\right)}{2} + C$

$C = 0 \quad x \ln^3 \frac{\sqrt{3}}{2} = -\frac{1}{2} \ln^2 \frac{\sqrt{3}}{2} + 0$

$y = \frac{\pi}{6}$

$x = -\frac{1}{2 \ln\left(\frac{\sqrt{3}}{2}\right)}$

$x = \frac{1}{\ln \frac{4}{3}} = \frac{1}{\ln 4 - \ln 3}$

$mn = 12$

Question32

Let f be a differentiable function such that $x^2 f(x) - x = 4 \int_0^x t f(t) dt$, $f(1) = \frac{2}{3}$.. Then 18f(3) is equal to : [10-Apr-2023 shift 1]

Options:

A. 180

B. 150

C. 210

D. 160

Answer: D

Solution:

Solution:

$x^2 f(x) - x = 4 \int_0^x t f(t) dt$

Differentiate w.r.t. x

$x^2 f'(x) + 2x f(x) - 1 = 4x f(x)$

Let $y = f(x)$

$\Rightarrow x^2 \frac{dy}{dx} - 2xy - 1 = 0$

$\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2}$

I.F. $= e^{\int \frac{-2}{x} dx} = \frac{1}{x^2}$

Its solution is

$\frac{y}{x^2} = \int \frac{1}{x^4} dx + C$

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$\because f(1)=\frac{2}{3} \Rightarrow y(1)=\frac{2}{3}$
$\Rightarrow \frac{2}{3}=-\frac{1}{3}+C$
$\Rightarrow C=1$
$\because y=-\frac{1}{3x}+x^2$
$f(x)=-\frac{1}{3x}+x^2$
$f(3)=-\frac{1}{9}+9=\frac{80}{9} \Rightarrow 18f(3)=160$

Question33

The slope of tangent at any point (x, y) on a curve y = y(x) is $\frac{x^2+y^2}{2xy}$, x > 0. If y(2) = 0, then a value of y(8) is:
[10-Apr-2023 shift 1]

Options:

A. $4\sqrt{3}$

B. $-4\sqrt{2}$

C. $-2\sqrt{3}$

D. $2\sqrt{3}$

Answer: A

Solution:

Solution:
$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$
$y=vx$
$y(2)=0$
$y(8)=?$
$\frac{dv}{dx}=v+x\frac{dv}{dx}$
$v+\frac{xdv}{dx}=\frac{x^2+v^2x^2}{2vx^2}$
$x\cdot\frac{dv}{dx}=\left(\frac{v^2+1}{2v}-v\right)$
$\frac{2vdv}{(1-v^2)}=\frac{dx}{x}$
$-\ln(1-v^2)=\ln x+C$
$\ln x+\ln(1-v^2)=C$
$\ln\left[x\left(1-\frac{y^2}{x^2}\right)\right]=C$
$\ln\left[\left(\frac{x^2-y^2}{x}\right)\right]=C$
$x^2-y^2=cx$
$y(2)=0$ at $x=2, y=0$
$4=2C \Rightarrow C=2$
$x^2-y^2=2x$
Hence, at $x=8$
$64-y^2=16$

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Option (1)

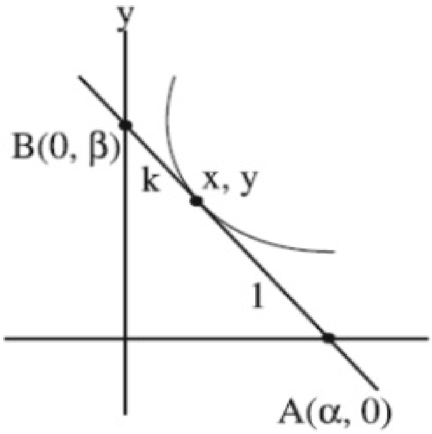
Question34

Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x axis and y-axis at the points A and B respectively. If PA : PB = 1 : k and y = y(x) is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, y(0) = k, then 4y then $4y(1) - 5\log_e 3$ is equal to ________.
[10-Apr-2023 shift 2]

Answer: 5

Solution:

$Y - y = \frac{dy}{dx}(X - x)$
$Y = 0$
$X = \frac{-y\,dx}{dy} + x$



$\frac{k\alpha + 0}{k+1} = x,\ \alpha = \frac{k+1}{k}x$
$\frac{k+1}{k}x = -y\frac{dx}{dy} + x$
$x + \frac{x}{k} = -y\frac{dx}{dy} + x$
$x\frac{dy}{dx} + ky = 0$
$\frac{dy}{dx} + \frac{k}{x}y = 0$
$y \cdot x^k = C$
$C = 1$
$100 \cdot \left(\frac{1}{10}\right)^k = 1$
$K = 2$
$\frac{dy}{dx} = \ln(2x + 1)$
$y = \frac{(2x+1)}{2}(\ln(2x+1) - 1) + c$
$2 = \frac{1}{2}(0 - 1) + C$

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$C = 2 + \frac{1}{2} = \frac{5}{2}$

$y(1) = \frac{3}{2}(\ell \ln 3 - 1) + \frac{5}{2}$

$= \frac{3}{2}\ln 3 + 1$

$4y(1) = 6\ln 3 + 4$

$4y(1) - 5\ln 3 = 4 + \ln 3$

Question35

Let $y = y(x)$ be a solution curve of the differential equation. $(1 - x^2y^2)\,dx = y\,dx + x\,dy$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of α is :
[11-Apr-2023 shift 1]

Options:

A. $\frac{1 + 3e^2}{2(3e^2 - 1)}$

B. $\frac{1 - 3e^2}{2(3e^2 + 1)}$

C. $\frac{3e^2}{2(3e^2 - 1)}$

D. $\frac{3e^2}{2(3e^2 + 1)}$

Answer: A

Solution:

Solution:

$(1 - x^2y^2)\,dx = y\,dx + x\,dy,\ y(1) = 2$

$y(2) = \infty$

$d\,x = \frac{d\,(xy)}{1 - (xy)^2}$

$\int dx = \int \frac{d\,(xy)}{1 - (xy)^2}$

$x = \frac{1}{2}\ln\left|\frac{1 + xy}{1 - xy}\right| + C$

Put $x = 1$ and $y = 2$:

$1 = \frac{1}{2}\ln\left|\frac{1 + 2}{1 - 2}\right| + C$

$C = 1 - \frac{1}{2}\ln 3$

Now put $x = 2$:

$2 = \frac{1}{2}\ln\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| + 1 - \frac{1}{2}\ln 3$

$1 + \frac{1}{2}\ln 3 = \frac{1}{2}\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right|$

$2 + \ln 3 = \left|\frac{1 + 2\alpha}{1 - 2\alpha}\right|$

$\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| = 3e^2$

$\frac{1 + 2\alpha}{1 - 2\alpha} = 3e^2, -3e^2$

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$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \Rightarrow \alpha = \frac{3e^2-1}{2(3e^2+1)}$

And $\frac{1+2\alpha}{1-2\alpha} = -3e^2 \Rightarrow \alpha = \frac{3e^2+1}{2(3e^2-1)}$

Question36

Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}$, x > 0. If y(1) = 2, then y(2) is equal to [11-Apr-2023 shift 2]

Options:

A. $\frac{693}{128}$

B. $\frac{637}{128}$

C. $\frac{697}{128}$

D. $\frac{679}{128}$

Answer: A

Solution:

Solution:

$\text{I.F} = = e^{\int \frac{5\,dx}{x(x^5+1)}} = e^{e^{\frac{5x^{-6}-6x}{(x^{-5}+1)}}}$

Put, $1+x^{-5} = t \Rightarrow -5x^{-6}dx = dt$

$\Rightarrow e^{\int \frac{-dt}{1}} = \frac{1}{t} = \frac{x^5}{1+x^5}$

$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{(1+x^5)} \times \frac{(1+x^5)^2}{x^7}dx$

$= \int x^3 dx + \int x^{-2} dx$

$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$

Given than: $x = 1 \Rightarrow y = 2$

$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$

$c = \frac{7}{4}$

$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$

Now put, $x = 2$

$y \cdot \left(\frac{32}{33}\right) = \frac{21}{4}$

$y = \frac{693}{128}$

Question37

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Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)dy = y(x - y)dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then [12-Apr-2023 shift 1]

Options:

A. $e^{3\beta^{-1}} = e(5 + \sqrt{2})$

B. $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$

C. $e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2})$

D. $e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$

Answer: B

Solution:

Solution:

$(1 + x^2)dy = y(x - y)dx$

$Y(0) = 1 \cdot y(2\sqrt{2}) = \beta$

$\frac{dy}{dx} = \frac{yx - y^2}{1 + x^2}$

$\frac{dy}{dx} + y\left(\frac{-x}{1 + x^2}\right) = \left(\frac{-1}{1 + x^2}\right)y^2$

$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{-x}{1 + x^2}\right) = \frac{-1}{1 + x^2}$

Put $\frac{1}{y} = t$ then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\frac{x}{1 + x^2} = \frac{1}{1 + x^2}$

If $= e^{\int \frac{x}{1 + x^2}dx} = e^{\frac{1}{2}\ln(1 + x^2)} \sqrt{1 + x^2}$

$t\sqrt{1 + x^2} = \int \frac{1}{\sqrt{1 + x^2}}dx$

$\frac{\sqrt{1 + x^2}}{y} = \ln\left(x + \sqrt{x^2 + 1}\right) + c$

$y(0) = 1 \Rightarrow c = 1$

$\Rightarrow \sqrt{1 + x^2} = y\ln\left(e\left(x + \sqrt{x^2 + 1}\right)\right)$

$\beta = \frac{3}{\ln(e(3 + 2\sqrt{2}))} \Rightarrow \frac{3}{\beta} = \ln(e(3 + 2\sqrt{2}))$

$e^{\frac{3}{\beta}} = e(3 + 2\sqrt{2})$

Question38

Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at [13-Apr-2023 shift 1]

Options:

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B. infinite number of points

C. one point

D. two points

Answer: A

Solution:

Solution:

$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$

I.F. $= e^{-x}$

$ye^{-x} = \int 7e^{-x} dx$

$\Rightarrow ye^{-x} = -7e^{-x} + c$

$\Rightarrow y = -7 + ce^{x}$

$-7 + 7e^{x} = -7 + 8e^{x} \Rightarrow e^{x} = 0$

No solution

Question39

If y = y(x) is the solution of the differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, x > 1 such that $y(2) = \frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2}) = \alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}$, $\alpha, \beta, \gamma, \in N$, then $\alpha\beta\gamma$ is equal to ______ [13-Apr-2023 shift 2]

Answer: 6

Solution:

given differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{5/2}}$ is linear D.E.

I.F. $= e^{\int \frac{4x}{x^2-1}} dx = e_e 2\ln(x^2-1) = e_e \ln(x^2-1)^2 = (x^2-1)^2$

$y(x^2-1)^2 = \int \frac{x+2}{(x^2-1)^{5/2}}(x^2-1)^2 dx$

$= \int \frac{x}{\sqrt{x^2-1}} dx + \int \frac{2\,dx}{\sqrt{x^2-1}}$

$= \sqrt{x^2-1} + 2\ln\left[x+\sqrt{x^2-1}\right] + C$

put $y(2) = \frac{2}{9}\ln(2+\sqrt{3})$

$\frac{2}{9}\ln(2+\sqrt{3})(9) = \sqrt{3} + 2\ln[2+\sqrt{3}] + C$

$= C = -\sqrt{3}$

put $x = \sqrt{2}$

$y = 1 + 2\ln[\sqrt{2}+1] - \sqrt{3}$

$\alpha = 2, \beta = 1 = \gamma = 3$

$\alpha\beta\gamma = 2(1)(3) = 6$

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Question40

Let x = x(y) be the solution of the differential equation $2(y+2)\log_e(y+2)\,dx + (x+4-2\log_e(y+2))\,dy = 0$, $y > -1$ with $x(e^4-2) = 1$. Then $x(e^9-2)$ is equal to
[15-Apr-2023 shift 1]

Options:

A. $\frac{4}{9}$

B. $\frac{32}{9}$

C. $\frac{10}{3}$

D. 3

Answer: B

Solution:

Solution:

$2(y+2)\ln(y+2)\,dx + (x+4-2\ln(y+2))\,dy = 0$

$2\ln(y+2) + (x+4-2\ln(y+2))\frac{1}{y+2}\cdot\frac{dy}{dx} = 0$

let, $\ln(y+2) = t$

$\frac{1}{y+2}\cdot\frac{dy}{dx} = \frac{dt}{dx}$

$2t + (x+4-2t)\cdot\frac{dt}{dx} = 0$

$(x+4-2t)\frac{dt}{dx} = -2t$

$\frac{dx}{dt} = \frac{2t-4-x}{2t}$

$\frac{dx}{dt} + \frac{x}{2t} = \frac{2t-4}{2t}$

$x\cdot t^{1/2} = \int\frac{2t-4}{2t}\cdot t^{1/2}\cdot dt$

$x\cdot t^{1/2} = \int\left(t^{1/2} - \frac{2}{t^{1/2}}\right)\cdot dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$x\cdot t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$

$x = \frac{2}{3}\cdot t - 4 + C\cdot t^{\frac{-1}{2}}$

$x = \frac{2}{3}\ln(y+2) - 4 + C\cdot(\ln(y+2))^{\frac{-1}{2}}$

Put $y = e^4 - 2$, $x = 1$

$1 = \frac{2}{3}\times 4 - 4 + C\times\frac{1}{2}$

$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$

$\therefore C = \frac{14}{\ldots}$

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$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$

$= 2 + \frac{14}{9}$

$= 3$

Question41

The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\frac{x^2}{xy - x^2y^2 - 1}$. If the curve passes through the point (1, 1), then e. y(e) is equal to
[24-Jun-2022-Shift-2]

Options:

A. $\frac{1 - \tan(1)}{1 + \tan(1)}$

B. $\tan(1)$

C. 1

D. $\frac{1 + \tan(1)}{1 - \tan(1)}$

Answer: D

Solution:

Solution:

$\because -\frac{dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$\therefore \frac{dy}{dx} = \frac{x^2y^2 - xy + 1}{x^2}$

Let $xy = v \Rightarrow y + x\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dv}{dx} - y = \frac{(v^2 - v + 1)y}{v}$

$\therefore \frac{dv}{dx} = \frac{v^2 + 1}{x}$

$\because y(1) = 1 \Rightarrow \tan^{-1}(xy) = \ln x + \tan^{-1}(1)$

Put $x = e$ and $y = y(e)$ we get

$\tan^{-1}(e \cdot y(e)) = 1 + \tan^{-1}1$

$\tan^{-1}(e \cdot y(e)) - \tan^{-1}1 = 1$

$\therefore e(y(e)) = \frac{1 + \tan(1)}{1 - \tan(1)}$

Question42

Let y = y(x) be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the

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curve y = y(x) is :
[25-Jun-2022-Shift-1]

Options:

A. not a critical point

B. a point of local minima

C. a point of local maxima

D. a point of inflection

Answer: B

Solution:

Solution:

$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$

If $e^{-\int \frac{1}{x+1}x} = e^{-\log(x+1)} = \frac{1}{x+1}$

$\therefore y\left(\frac{1}{x+1}\right) = \int \frac{e^{3x}(x+1)}{x+1}dx$

$\frac{y}{x+1} = \int e^{3x}dx$

$\frac{y}{x+1} = \frac{e^{3x}}{3} + c$

$\because y(0) = \frac{1}{3}$

$\frac{1}{3} = \frac{1}{3} + c$

$\therefore c = 0$

So : $y = \frac{e^{3x}}{3}(x+1)$

$y' = e^{3x}(x+1) + \frac{e^{3x}}{3} = e^{3x}\left(x + \frac{4}{3}\right)$

$y'' = 3e^{3x}\left(x + \frac{4}{3}\right) + e^{3x} = e^{3x}(3x+5)$

$y' = 0$ at $x = \frac{-4}{3}$ & $y'' = e^{-4}(1) > 0$ at $x = \frac{-4}{3}$

$\Rightarrow x = \frac{-4}{3}$ is point of local minima

Question43

If the solution curve y = y(x) of the differential equation $y^2 dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to :
[25-Jun-2022-Shift-1]

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Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

Solution:

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2 - y^2}$$

Put $y = vx$ we get

$$v + x\frac{dv}{dx} = \frac{v^2}{v - 1 - v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - v^2 + v + v^3}{v - 1 - v^2}$$

$$\Rightarrow \int \frac{v - 1 - v^2}{v(1 + v^2)} dv = \int \frac{dx}{x}$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x}\right) = \ln x + c$$

As it passes through $(1, 1)$

$$c = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) \ln\left(\frac{y}{x}\right) = \ln x + \frac{\pi}{4}$$

Put $y = \sqrt{3}x$ we get

$$\Rightarrow \frac{\pi}{3} - \ln\sqrt{3} = \ln x + \frac{\pi}{4}$$

$$\Rightarrow \ln x = \frac{\pi}{12} - \ln\sqrt{3} = \ln\alpha$$

$$\therefore \ln(\sqrt{3}\alpha) = \ln\sqrt{3} + \ln\alpha$$

$$= \ln\sqrt{3} + \frac{\pi}{12} - \ln\sqrt{3} = \frac{\pi}{12}$$

Question44

If $y = y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then $y(1)$ is equal to [25-Jun-2022-Shift-2]

Options:

A. $\frac{1}{3}$

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B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

Answer: B

Solution:

Solution:

$2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$

$\Rightarrow 2x(xdy - ydx) + 3y^3 dx = 0$

$\Rightarrow 2\left(\frac{xdy - ydx}{y^2}\right) + 3\frac{dx}{x} = 0$

$\Rightarrow -\frac{2x}{y} + 3\ln x = C$

$\because y(e) = \frac{e}{3} \Rightarrow -6 + 3 = C \Rightarrow C = -3$

Now, at $x = 1$, $-\frac{2}{y} + 0 = -3$

$y = \frac{2}{3}$

Question45

Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then y(2) is equal to____
[26-Jun-2022-Shift-1]

Answer: 12

Solution:

$(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$

$\Rightarrow \frac{dy}{dx} = \left(\frac{6x}{x^2 + 4}\right)y + 2x$

$\Rightarrow \frac{dy}{dx} - \left(\frac{6x}{x^2 + 4}\right)y = 2x$

I. F. $= e^{-3\ln(x^2+4)} = \frac{1}{(x^2+4)^3}$

So $\frac{y}{(x^2+4)^3} = \int \frac{2x}{(x^2+4)^3} dx + c$

$\Rightarrow y = -\frac{1}{2}(x^2 + 4) + c(x^2 + 4)^3$

When $x = 0$, $y = 0$ gives $c = \frac{1}{32}$,

So, for $x = 2$, $y = 12$

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Question46

Let $S = (0, 2\pi) - \left\{ \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$. Let $y = y(x)$, $x \in S$, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$, $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve $y = y(x)$ with the curve $y = \sqrt{2} \sin x$ is $\frac{k\pi}{12}$, then k is equal to____
[26-Jun-2022-Shift-1]

Answer: 42

Solution:

$\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$

$\Rightarrow dy = \frac{\sec^2 x dx}{(1 + \tan x)^2}$

$\Rightarrow y = -\frac{1}{1 + \tan x} + c$

When $x = \frac{\pi}{4}$, $y = \frac{1}{2}$ gives c = 1

So $y = \frac{\tan x}{1 + \tan x} \Rightarrow y = \frac{\sin x}{\sin x + \cos x}$

Now, $y = \sqrt{2} \sin x \Rightarrow \sin x = 0$

or $\sin x + \cos x = \frac{1}{\sqrt{2}}$

$\sin x = 0$ gives $x = \pi$ only.

and $\sin x + \cos x = \frac{1}{\sqrt{2}} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$

So $x + \frac{\pi}{4} = \frac{5\pi}{6}$ or $\frac{13\pi}{6} \Rightarrow x = \frac{7\pi}{12}$ or $\frac{23\pi}{12}$

Sum of all solutions $= \pi + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{42\pi}{12}$

Hence, $k = 42$.

Question47

If the solution of the differential equation $\frac{dy}{dx} + e^x(x^2 - 2)y = (x^2 - 2x)(x^2 - 2)e^{2x}$ satisfies $y(0) = 0$, then the value of $y(2)$ is__
[26-Jun-2022-Shift-2]

Options:

A. -1

B. 1

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C. 0

D. e

Answer: C

Solution:

$\because \frac{dy}{dx} + e^x(x^2-2)y = (x^2-2x)(x^2-2)e^{2x}$

Here, $I.F. = e^{\int e^x(x^2-2)dx}$

$= e^{(x^2-2x)e^x}$

$\therefore$ Solution of the differential equation is

$y \cdot e^{(x^2-2x)e^x} = \int (x^2-2x)(x^2-2)e^{2x} \cdot e^{(x^2-2x)e^x} dx$

$= \int (x^2-2x)e^x \cdot (x^2-2)e^x \cdot e^{(x^2-2x)e^x} dx$

Let $(x^2-2x)e^x = t$

$\therefore (x^2-2)e^x dx = dt$

$y \cdot e^{(x^2-2x)e^x} = \int t \cdot e^t dt$

$y \cdot e^{(x^2-2x)e^x} = (x^2-2x-1)e^{(x^2-2x)e^x} + c$

$\therefore y(0) = 0$

$\therefore c = 1$

$\therefore y = (x^2-2x-1) + e^{(2x-x^2)e^x}$

$\therefore y(2) = -1+1 = 0$

Question48

Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is :
[27-Jun-2022-Shift-1]

Options:

A. 10

B. 8

C. 7

D. 5

Answer: B

Solution:

$\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$
$= bxdy + cydy + ady = axdx - bydx + adx$
$= cydy + ady - axdx - adx + b(xdy + ydx) = 0$
$= c\int ydy + a\int xdx - a\int dx + b\int d(xy) = 0$
$= \frac{cy^2}{2} + ay - \frac{ax^2}{2} - ax + bxy = k$
$= ax^2 - cy^2 + 2ax - 2ay - 2bxy = k$
Above equation is circle
$\Rightarrow a = -c$ and $b = 0$
$ax^2 + ay^2 + 2ax - 2ay = k$
$\Rightarrow x^2 + y^2 + 2x - 2y = \lambda \ \left[\lambda = \frac{k}{a}\right]$
Passes through (2, 5)

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Circle $\equiv x^2 + y^2 + 2x - 2y - 23 = 0$
Centre $(-1, 1) r = \sqrt{(-1)^2 + 1^2 + 23} = 5$
Shortest distance of $(11, 6) = \sqrt{12^2 + 5^2} - 5$
$= 13 - 5$
$= 8$

Question49

If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$, then $y(2)$ is equal to : [27-Jun-2022-Shift-1]

Options:

A. $2 + \log_2 3$

B. $2 + \log_3 2$

C. $2 - \log_3 2$

D. $2 - \log_2 3$

Answer: D

Solution:

Solution:
$\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$
$\frac{dy}{dx} = -\frac{2^x(2^y - 1)}{2^y(2^z - 1)}$
$\int \frac{2^y}{2^y - 1} dy = -\int \frac{2^x}{2^x - 1} dx$
$= \frac{\log_e(2^y - 1)}{\log_{e^2}} = -\frac{\log_e(2^x - 1)}{\log_e 2} + \frac{\log_e c}{\log_e 2}$
$= |(2^y - 1)(2^x - 1)| = c$
$\because y(1) = 1$
$\therefore c = 1$
$= |(2^y - 1)(2^x - 1)| = 1$
For $x = 2$
$|(2^y - 1)3| = 1$
$2^y - 1 = \frac{1}{3} \Rightarrow 2y = \frac{4}{3}$
Taking log to base 2.
$\therefore y = 2 - \log_2 3$

Question50

If the solution curve of the differential equation $((\tan^{-1}y) - x)dy = (1 + y^2)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is [27-Jun-2022-Shift-2]

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A. 2e

B. $\frac{2}{e}$

C. 2

D. $\frac{1}{e}$

Answer: B

Solution:

Solution:

$((\tan^{-1}y) - x)dy = (1+y^2)dx$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

$\text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution

$x.\ e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}\tan^{-1}y}{1+y^2}dy$

Let $e^{\tan^{-1}y} = t$

$\frac{e^{\tan-1}y}{1+y^2} = dt$

$= xe^{\tan^{-1}y} = \int \ln t\, dt = t\ln t - t + c$

$\therefore = xe^{\tan^{-1}y} = e^{\tan^{-1}y}\tan^{-1}y - e^{\tan^{-1}y} + c\ldots..\ (i)$

$\because$ It passes through $(1, 0) \Rightarrow c = 2$

Now put $y = \tan 1$, then

$ex = e - e + 2$

$$

$\Rightarrow x = \frac{2}{e}$

Question51

Let y = y(x) be the solution of the differential equation $(1-x^2)dy = \left(xy + (x^3+2)\sqrt{1-x^2}\right)dx$, $-1 < x < 1$, and $y(0) = 0$. If $\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}y(x)dx = k$, then k^{-1} is equal to____

[27-Jun-2022-Shift-2]

Answer: 320

Solution:

Solution:

$(1-x^2)dy = \left(xy + (x^3+2)\sqrt{1-x^2}\right)dx$

$\therefore \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{x^3+3}{\sqrt{1-x^2}}$

$\therefore$ I. F. $= e^{\int -\frac{x}{1-x^2}dx} = \sqrt{1-x^2}$

Solution is

$y.\sqrt{1-x^2} = \int (x^3+3)\,dx$

$y \cdot \sqrt{1-x^2} = \frac{x^4}{4} + 3x + c$

$\because y(0) = 0 \Rightarrow c = 0$

$\therefore y(x) = \frac{x^4+12x}{4\sqrt{1-x^2}}$

$\therefore \int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}y(x)\,dx = \int_{\frac{-1}{2}}^{\frac{1}{2}} \left(\frac{x^4+12x}{4}\right) dx = \int_0^{\frac{1}{2}} \frac{x^4}{2}dx$

$\therefore k = \frac{1}{320}$

$\therefore = k^{-1} = 320$

Question52

Let the solution curve y = y(x) of the differential equation

$$\left[\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right] x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right] y$$

pass through the points (1, 0) and (2α, α), α > 0. Then α is equal to [28-Jun-2022-Shift-1]

Options:

A. $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$

B. $\frac{1}{2}\exp\left(\frac{\pi}{6} + e - 1\right)$

C. $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$

D. $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

Answer: A

Solution:

Solution:

$$\left(\frac{1}{\sqrt{1-\frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = 1 + \left(\frac{1}{\sqrt{1-\frac{y^2}{x^2}}} + e^{\frac{y}{x}}\right)\frac{y}{x}$$

Putting $y = tx$

$$\left(\frac{1}{\sqrt{1-t^2}} + e^t\right)\left(t + x\frac{dt}{dx}\right) = 1 + \left(\frac{1}{\sqrt{1-t^2}} + e^t\right)t$$

$$\Rightarrow x\left(\frac{1}{\sqrt{1-t^2}} + e^t\right)\frac{dt}{dx} = 1$$

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$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + C$

at $x = 1, y = 0$

So, $0 + e^{0} = 0 + C \Rightarrow C = 1$

at $(2\alpha, \alpha)$

$\sin - 1\left(\frac{y}{x}\right) + e^{y/x} = \ln x + 1$

$\Rightarrow \frac{\pi}{6} + e^{\frac{1}{2}} - 1 = \ln(2\alpha)$

$\Rightarrow \alpha = \frac{1}{2} e^{\left(\frac{\pi}{6} + e^{\frac{1}{2}} - 1\right)}$

Question53

Let $y = y(x)$ be the solution of the differential equation $x(1 - x^2)\frac{dy}{dx} + (3x^2y - y - 4x^3) = 0$, $x > 1$, with $y(2) = -2$. Then $y(3)$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. -18

B. -12

C. -6

D. -3

Answer: A

Solution:

Solution:

$\frac{dy}{dx} + \frac{y(3x^2 - 1)}{x(1 - x^2)} = \frac{4x^3}{x(1 - x^2)}$

$IF = e^{\int \frac{3x^2 - 1}{x - x^3} dx} = e^{-\ln\left|x^3 - x\right|} = e^{-\ln(x^3 - x)} = \frac{1}{x^3 - x}$

Solution of D.E. can be given by

$y . \frac{1}{x^3 - x} = \int \frac{4x^3}{x(1 - x^2)} \cdot \frac{1}{x(x^2 - 1)} dx$

$\Rightarrow \frac{y}{x^3 - x} = \int \frac{-4x}{(x^2 - 1)^2} dx$

$\Rightarrow \frac{y}{x^3 - x} = \frac{2}{(x^2 - 1)} + c$

at $x = 2, y = -2$

$\frac{-2}{6} = \frac{2}{3} + c \Rightarrow c = -1$

at $x = 3 \Rightarrow \frac{y}{24} = \frac{2}{8} - 1 \Rightarrow y = -18$

Question54

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$2ye^{x/y^2}dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to :
[28-Jun-2022-Shift-2]

Options:

A. $e\log_e(2)$

B. $-\log_e(2)$

C. $e^2\log_e(2)$

D. $-e^2\log_e(2)$

Answer: D

Solution:

Solution:
Given differential equation

$2ye^{\frac{x}{y^2}}dx + \left(y^2 - 4xe^{\frac{x}{y^2}}\right)dy = 0,\ x(1) = 0$

$\Rightarrow e^{\frac{x}{y^2}}[2y\,dx - 4x\,dy] = -y^2dy$

$\Rightarrow e^{\frac{x}{y^2}}\left[\frac{2y^2dx - 4xy\,dy}{y^4}\right] = \frac{-1}{y}dy$

$\Rightarrow 2e^{\frac{x}{y^2}}d\left(\frac{x}{y^2}\right) = -\frac{1}{y}dy$

$\Rightarrow 2e^{\frac{x}{y^2}} = -\ln y + c.....$ (i)
Now, using $x(1) = 0$, $c = 2$
So, for $x(e)$, Put $y = e$ in (i)

$2e^{\frac{x}{e^2}} = -1 + 2$

$\Rightarrow \frac{}{e^2} = \ln\left(\frac{1}{2}\right) \Rightarrow x(e) = -e^2\ln 2$

Question55

Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 15

B. 11

C. 13

D. 17

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Solution:

Solution:
Given,
$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x}$
$\Rightarrow x\frac{dy}{dx} = y + \sqrt{y^2 + 16x}$
$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 16}$
This is a homogenous different equation.
Let $\frac{y}{x} = v$
$\Rightarrow y = vx$
$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\therefore v + x\frac{dv}{dx} = v + \sqrt{v^2 + 16}$
$\Rightarrow x\frac{dv}{dx} = \sqrt{v^2 + 16}$
$\Rightarrow \frac{dv}{\sqrt{v^2 + 16}} = \frac{dx}{x}$
Integrating both sides, we get
$\int \frac{dv}{\sqrt{v^2 + 16}} = \int \frac{dx}{x}$
$\Rightarrow \ln\left|v + \sqrt{v^2 + 16}\cdot\right| = \ln x + \ln c$
$\Rightarrow v + \sqrt{v^2 + 16} = cx$
Now putting, $v = \frac{y}{x}$, we get
$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} + 16} = cx$
$\Rightarrow \frac{y}{x} + \sqrt{\frac{y^2 + 16x^2}{x^2}} = cx$
$\Rightarrow y + \sqrt{y^2 + 16x^2} = cx^2$......
Given, $y(1) = 3$
$\therefore$ When $x = 1$ then $y = 3$.
Putting in equation (1) we get,
$3 + \sqrt{9 + 16} = c.1$
$\Rightarrow c = 8$
$\therefore$ Solution of equation,
$y + \sqrt{y^2 + 16x^2} = 8x^2$
Now, $y(2)$ means when $x = 2$ then $y = ?$

$\therefore y + \sqrt{y^2 + 16 \times 4} = 8 \times 4$
$\Rightarrow y = 15$

Question56

Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4x - \cos^2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$, $0 < x < \frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$ If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to____
[29-Jun-2022-Shift-1]

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Answer: 2

Solution:

Solution:

$$\frac{dy}{dx}+\frac{\sqrt{2}}{2\cos^4x-\cos 2x}y=xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$\int\frac{dx}{2\cos^4x-\cos 2x}$$

$$=\int\frac{dx}{\cos^4x+\sin^4x}=\int\frac{\operatorname{cosec}^4x\,dx}{1+\cot^4x}$$

$$=-\int\frac{t^2+1}{t^4+1}dt=-\int\frac{1}{\left(t-\frac{1}{t}\right)^2+2}dt=\frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)$$

$\operatorname{Cot}x=t$

$$=-\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\cot 2x)$$

$$\therefore IF=e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)}=\int x dx$$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)}=\frac{x^2}{2}+c$$

$$y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{32}+c\Rightarrow c=0$$

$$y=\frac{x^2}{2}e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{18}e^{\tan^{-1}\left(\sqrt{2}\cot\frac{2\pi}{3}\right)}$$

$$=\frac{\pi^2}{18}e^{-\tan^{-1}}\left(\sqrt{\frac{2}{3}}\right)$$

$$\alpha=\sqrt{\frac{2}{3}}\Rightarrow 3\alpha^2=2$$

Question57

If $y=y(x)$ is the solution of the differential equation $(1+e^{2x})\frac{dy}{dx}+2(1+y^2)e^x=0$ and $y(0)=0$ then $6(y'(0)+(y(\log_e\sqrt{3}))^2)$ is equal to
[29-Jun-2022-Shift-2]

Options:

A. 2

B. -2

C. -4

D. -1

Answer: C

Solution:

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$\frac{dy}{1+y^2} + \frac{2e^x}{1+e^{2x}} dx = 0$
on integration
$\tan^{-1}y + 2\tan^{-1}e^x = c$
$\because y(0) = 0$
so, $C = \frac{\pi}{2} \Rightarrow \tan^{-1}y + 2\tan^{-1}e^x = \frac{\pi}{4}$ from eq.(i), $\left(\frac{dy}{dx}\right)_{x=0} = -1$ arg $y(\ln\sqrt{3}) = -\frac{1}{\sqrt{3}}$
$6\left[y'(0) + (y(\ln\sqrt{3})^2]\right. = 6\left[-1 + \frac{1}{3}\right] = -4.$

Question58

Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^{\alpha}+1}{\beta e^{\alpha}}$, then the value of $\alpha + \beta$ is equal to____
[29-Jun-2022-Shift-2]

Answer: 14

Solution:

Solution:
$\frac{dy}{dx} + \frac{2x}{x-1} \cdot y = \frac{1}{(x-1)^2}$
$y = \frac{1}{(x-1)^2}\left[\frac{e^{2x}+1}{2e^{2x}}\right]$
$y(3) = \frac{e^6+1}{8e^6}$
$\alpha + \beta = 14$

Question59

If $x = x(y)$ is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$; then $x(e)$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. $e^3(e^e - 1)$

B. $e^e(e^3 - 1)$

C. $e^2(e^e + 1)$

D. $e^e(e^2 - 1)$

Answer: A

Solution:

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$$\frac{dx}{dy} - \frac{2x}{y} = y^2(y+1)e^y$$

$$\text{If } = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

Solution is given by

$$x \cdot \frac{1}{y^2} = \int y^2(y+1)e^y \cdot \frac{1}{y^2}dy$$

$$\Rightarrow \frac{x}{y^2} = \int (y+1)e^y dy$$

$$\Rightarrow \frac{x}{y^2} = ye^y + c$$

$$\Rightarrow x = y^2(ye^y + c) \text{ at, } y = 1, x = 0$$

$$\Rightarrow 0 = 1(1 \cdot e^1 + c) \Rightarrow c = -e \text{ at } y = e$$

$$x = e^2(e \cdot e^e - e)$$

Question60

The slope of the tangent to a curve C : y = y(x) at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to :
[25-Jul-2022-Shift-1]

Options:

A. $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

B. $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

C. $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

D. $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Answer: B

Solution:

Solution:

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$$\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}=e^{2x}-\frac{6e^{-x}}{2+9e^{-2x}}$$

$$\int dy=\int e^{2x}dx-3\int 1+\left(\frac{3e^{-x}}{\sqrt{2}}\right)^{2}dx(\text{ put } e^{-x}=t)$$

$$=\frac{e^{2x}}{2}+3\int\frac{dt}{1+\left(\frac{3t}{\sqrt{2}}\right)^{2}}$$

$$=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\frac{3t}{\sqrt{2}}+C$$

$$y=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-x}}{\sqrt{2}}\right)+C$$

It is given that the curve passes through

$$\left(0,\ \frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$$

$$\frac{1}{2}+\frac{\pi}{2\sqrt{2}}=\frac{1}{2}+\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)+C$$

$$\Rightarrow C=\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

Now if $\left(\alpha,\ \frac{1}{2}e^{2\alpha}\right)$ satisfies the curve, then

$$\frac{1}{2}e^{2\alpha}=\frac{e^{2\alpha}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)+\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

$$\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)=\frac{\pi}{2\sqrt{2}}\times\frac{1}{\sqrt{2}}=\frac{\pi}{4}$$

$$\frac{\frac{3}{\sqrt{2}}-\frac{3e^{-\alpha}}{\sqrt{2}}}{1+\frac{9}{2}e^{-\alpha}}=1$$

$$\frac{3}{\sqrt{2}}e^{\alpha}-\frac{3}{\sqrt{2}}=e^{\alpha}+\frac{9}{2}$$

$$e^{\alpha}=\frac{\frac{9}{2}+\frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}}-1}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$$

Question61

The general solution of the differential equation $(x-y^2)dx+y(5x+y^2)dy=0$ is: [25-Jul-2022-Shift-1]

Options:

A. $(y^2+x)^4=C\,|(y^2+2x)^3|$

B. $(y^2+2x)^4=C\,|(y^2+x)^3|$

C. $|(y^2+x)^3|=C(2y^2+x)^4$

D. $|(y^2+2x)^3|=C(2y^2+x)^4$

Answer: A

Solution:

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$(x-y^2)dx+y(5x+y^2)dy=0$

$y\frac{dy}{dx}=\frac{y^2-x}{5x+y^2}$

Let $y^2=t\ \frac{1}{2}\cdot\frac{dt}{dx}=\frac{t-x}{5x+t}$

Now substitute, $t=vx$

$\frac{dt}{dx}=v+x\frac{dv}{dx}$

$\frac{1}{2}\left\{v+x\frac{dv}{dx}\right\}=\frac{v-1}{5+v}$

$x\frac{dv}{dx}=\frac{2v-2}{5+v}-v=\frac{-3v-v^2-2}{5+v}$

$\int\frac{5+v}{v^2+3v+2}dv=\int-\frac{dx}{x}$

$\int\frac{4}{v+1}dv-\int\frac{3}{v+2}dv=-\int\frac{dx}{x}$

$4\ln|v+1|-3\ln|v+2|=-\ln x+\ln C$

$\left|\frac{(v+1)^4}{(v+2)^3}\right|=\frac{c}{x}$

$\left|\frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3}=\frac{c}{x}\right|$

$|(y^2+x)^4|=C\,|(y^2+2x)^3|$

Question62

Let a smooth curve y = f (x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to [25-Jul-2022-Shift-2]

Options:

A. $2\log_e 2$

B. 4

C. 1

D. $4\log_e 2$

Answer: B

Solution:

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$\frac{dy}{dx} \propto \frac{-y}{x}$

$\frac{dy}{dx} = \frac{-ky}{x} \Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$

$\ln|y| = -K \ln|x| + C$

If the above equation satisfy (1, 2) and (8, 1)

$\ln 2 = -K \times 0 + C \Rightarrow C = \ln 2$

$\ln 1 = -K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$

So, at $x = \frac{1}{8}$

$\ln|y| = -\frac{1}{3}\ln\left(\frac{1}{8}\right) + \ln 2 = 2\ln 2$

$|y| = 4$

Question63

Let a smooth curve y = f (x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to [25-Jul-2022-Shift-2]

Options:

A. $2\log_e 2$

B. 4

C. 1

D. $4\log_e 2$

Answer: B

Solution:

Solution:

$\frac{dy}{dx} \propto \frac{-y}{x}$

$\frac{dy}{dx} = \frac{-ky}{x} \Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$

$\ln|y| = -K \ln|x| + C$

If the above equation satisfy (1, 2) and (8, 1)

$\ln 2 = -K \times 0 + C \Rightarrow C = \ln 2$

$\ln 1 = -K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$

So, at $x = \frac{1}{8}$

$\ln|y| = -\frac{1}{3}\ln\left(\frac{1}{8}\right) + \ln 2 = 2\ln 2$

$|y| = 4$

Question64

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$\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}$, **y(1) = 1.**

If for some $n \in \mathbb{N}$, $y(2) \in [n - 1, n)$, then n is equal to __

[25-Jul-2022-Shift-2]

Answer: 3

Solution:

Solution:

$\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2 + 2x^2)}{(3y^2 + x^2)}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v(4v^2 + 2)}{(3v^2 + 1)}$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{(4v^2 + 2 - 3v^2 - 1)}{3v^2 + 1}\right)$

$\Rightarrow \int (3v^2 + 1)\frac{dv}{v^3 + v} = \int \frac{dx}{x}$

$\Rightarrow \ln|v^3 + v| = \ln x + c$

$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)\right| = \ln x + C$

$\downarrow y(1) = 1$

$\Rightarrow C = \ln 2$

$\therefore$ for $y(2)$

$\ln\left(\frac{y^3}{8} + \frac{y}{2}\right) = 2\ln 2 \Rightarrow \frac{y^3}{8} + \frac{y}{2} = 4$

$\Rightarrow [y(2)] = 2$

$\Rightarrow n = 3$

Question65

If $\frac{dy}{dx} + 2y\tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of y(x) is :

[26-Jul-2022-Shift-1]

Options:

A. $\frac{1}{8}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. $\frac{3}{8}$

Answer: A

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Solution:

$\frac{dy}{dx} + 2y\tan x = \sin x$

which is a first order linear differential equation.

Integrating factor (I. F.) $= e^{\int 2\tan x dx}$

$= e^{2\ln|\sec x|} = \sec^2 x$

Solution of differential equation can be written as

$y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x dx = \int \sec x\cdot\tan x dx\ y\sec^2 x = \sec x + C$

$y\left(\frac{\pi}{3}\right) = 0,\ 0 = \sec\frac{\pi}{3} + C \Rightarrow C = -2$

$y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$y_{max} = \frac{1}{8}$

Question66

Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 6

Solution:

$\int_3^x f(x)dx = \left(\frac{f(x)}{x}\right)^3$

$x^3\cdot\int_3^x f(x)dx = f^3(x)$

Differentiate w.r.t. x

$x^3 f(x) + 3x^2\cdot\frac{f^3(x)}{x^3} = 3f^2(x)f'(x)$

$\Rightarrow 3y^2\frac{dy}{dx} = x^3 y + \frac{3y^3}{x}$

$3xy\frac{dy}{dx} = x^4 + 3y^2$

Let $y^2 = t$

$\frac{3}{2}\frac{dt}{dx} = x^3 + \frac{3t}{x}$

$\frac{dt}{dx} - \frac{2t}{x} = \frac{2x^3}{3}$

I. F. $= .e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

Solution of differential equation

t. $\frac{1}{x^2} = \int\frac{2}{3}x dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

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$y^2 = \frac{x^4}{3} + Cx^2$
Curve passes through (3, 3) ⇒ C = −2

$y^2 = \frac{x^4}{3} - 2x^2$
Which passes through $(\alpha, 6\sqrt{10})$
$\frac{\alpha^4 - 6\alpha^2}{3} = 360$
$\alpha^4 - 6\alpha^2 - 1080 = 0$
$\alpha = 6$

Question67

Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$, $x \in (-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to

[26-Jul-2022-Shift-2]

Options:

A. $\frac{\pi}{3} - \frac{1}{4}$

B. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

C. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

D. $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Answer: B

Solution:

Solution:
$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$
which is first order linear differential equation.
Integrating factor (I . F) = $.e^{\int \frac{x}{x^2 - 1}dx}$
$= e^{\frac{1}{2}\ln|x^2 - 1|} = \sqrt{|x^2 - 1|}$
$= \sqrt{1 - x^2}$
$\because x \in (-1, 1)$
Solution of differential equation
$y\sqrt{1 - x^2} = \int (x^4 + 2x)dx = \frac{x^5}{5} + x^2 + c$
Curve is passing through origin, $c = 0$

$y = \frac{x^5 + 5x^2}{5\sqrt{1 - x^2}}$

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$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5 + 5x^2}{5\sqrt{1-x^2}} dx = 0 + 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$$

put $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$$I = 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta \cdot \cos\theta \, d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{3}} (1 - \cos 2\theta) d\theta$$

$$= \left(\theta - \frac{\sin 2\theta}{2}\right)\Bigg|_0^{\frac{\pi}{3}}$$

Question68

Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x \to \infty} y(x)$ is finite. If a and b are respectively the x - and y-intercepts of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to ________.
[26-Jul-2022-Shift-2]

Answer: 3

Solution:

Solution:

IF $= e^{-x}$

$y \cdot e^{-x} = -2e^{-x} + \frac{e^{-2x}}{2} + C$

$\Rightarrow y = -2 + e^{-x} + Ce^{x}$

$\lim_{x \to \infty} y(x)$ is finite so $C = 0$

$y = -2 + e^{-x}$

$\Rightarrow \frac{dy}{dx} = -.e^{-x} \Rightarrow \frac{dy}{dx}\Big|_{x=0} = -1$

Equation of tangent

$y + 1 = -1(x - 0)$

or $y + x = -1$

So $a = -1$, $b = -1$

$\Rightarrow a - 4b = 3$

Question69

Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is

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Options:

A. 0

B. 1

C. 2

D. 3

Answer: A

Solution:

$\frac{dy}{dx} = x + y$

Let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 1 = t \Rightarrow \int \frac{dt}{t+1} = \int dx$

$\ln | t + 1 | = x + C'$

$|t + 1| = Ce^x$

$|x + y + 1| = Ce^x$

For $y_1(x)$, $y_1(0) = 0 \Rightarrow C = 1$

For $y_2(x)$, $y_2(0) = 1 \Rightarrow C = 2$

$y_1(x)$ is given by $|x + y + 1| = e^x$

$y_2(x)$ is given by $|x + y + 1| = 2e^x$

At point of intersection

$e^x = 2e^x$

No solution

So, there is no point of intersection of $y_1(x)$ and $y_2(x)$.

Question70

Let $y = y(x)$ be the solution curve of the differential equation $\sin(2x^2)\log_e(\tan x^2)\,dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0$, $0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to ________.

[27-Jul-2022-Shift-1]

Answer: 1

Solution:

$$\frac{dy}{dx} + y\left(\frac{4x}{\sin(2x^2)\ln(\tan x^2)}\right) = \frac{4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)\ln(\tan x^2)}$$

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$\text{I . F} = e^{\int \frac{4x}{\sin(2x^2)\ln(\tan x^2)} dx}$

$= e^{\ln|\ln(\tan x^2)} = \ln(\tan x^2)$

$\therefore \int d\left(y \cdot \ln(\tan x^2)\right) = \int \frac{4\sqrt{2}x \sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)} dx$

$\Rightarrow y\ln(\tan x^2) = \ln\left|\frac{\sec x^2 + \tan x^2}{\operatorname{cosec} x^2 - \cot x^2}\right| + C$

$\ln\left(\frac{1}{\sqrt{3}}\right) = \ln\left(\frac{\frac{3}{\sqrt{3}}}{2-\sqrt{3}}\right) + C$

$e = \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$

For $y\left(\sqrt{\frac{\pi}{3}}\right)$

$y\ln(\sqrt{3}) = \ln\left|\frac{2+\sqrt{3}}{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}\right| + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$

$= \ln(2+\sqrt{3}) + \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$

$\Rightarrow y\ln\sqrt{3} = \ln\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \frac{y}{2}\ln 3 = -\frac{1}{2}\ln 3$

$\Rightarrow y = 1$

$\therefore \left|y\left(\sqrt{\frac{\pi}{3}}\right)\right| = 1$

Question71

Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$, $x > 0$, intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of α is :
[28-Jul-2022-Shift-1]

Options:

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $-\frac{3}{2}$

D. $\frac{5}{2}$

Answer: B

Solution:

Solution:

$\frac{xdy - ydx}{\sqrt{x^2 + y^2}} = dx$

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$\Rightarrow \frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$

Let $\frac{y}{x} = v$

$\Rightarrow v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$

OR $\ln\left(v + \sqrt{1 + v^2}\right) = \ln x + C$

at $x = 1, y = 0$

$\Rightarrow C = 0$

$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = x$

At $x = 2$

$\frac{y}{2} + \sqrt{1 + \frac{y^2}{4}} = 2$

$\Rightarrow 1 + \frac{y^2}{4} = 4 + \frac{y^2}{4} - 2y$

OR $y = \frac{3}{2}$

Question72

If $y = y(x), x \in (0, \pi / 2)$ be the solution curve of the differential equation $(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y - 2e^{-4x}(2\sin 2x + \cos 2x)$, with $y\left(\frac{\pi}{4}\right) = e^{-\pi}$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

[28-Jul-2022-Shift-1]

Options:

A. $\frac{2}{\sqrt{3}}e^{-2\pi / 3}$

B. $\frac{2}{\sqrt{3}}e^{2\pi / 3}$

C. $\frac{1}{\sqrt{3}}e^{-2\pi / 3}$

D. $\frac{1}{\sqrt{3}}e^{2\pi / 3}$

Answer: A

Solution:

Solution:

$(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y$

$= 2e^{-4x}(2\sin 2x + \cos 2x)$

$\frac{dy}{dx} + (8 + 4\cot 2x)y = 2e^{-4x}\left(\frac{2\sin 2x + \cos 2x}{\sin^2 2x}\right)$

Integrating factor

$(I.F.) = e^{\int(8 + 4\cot 2x)dx}$

$= e^{8x + 2\ln\sin 2x}$

Solution of differential equation

$y.e^{8x + 2\ln\sin 2x}$

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$= \int 2e^{(4x+2\ln\ \sin 2x)}\dfrac{(2\sin 2x+\cos 2x)}{\sin^2 2x}dx$

$= 2\int e^{4x}(2\sin 2x+\cos 2x)dx$

$y.\ e^{8x+2\ln\ \sin 2x} = e^{4x}\sin 2x + c$

$y\left(\frac{\pi}{4}\right) = e^{-\pi}$

$e^{-\pi}\cdot e^{2\pi} = e^{\pi} + c \Rightarrow c = 0$

$y\left(\frac{\pi}{6}\right) = \dfrac{e^{\frac{2\pi}{3}\frac{\sqrt{3}}{2}}}{e^{\left(\frac{4\pi}{3}+2\ln\frac{\sqrt{3}}{2}\right)}}$

$= e^{\frac{-2\pi}{3}}\cdot\dfrac{2}{\sqrt{3}}$

Question73

Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx} + \frac{1}{x^2-1}y = \left(\frac{x-1}{x+1}\right)^{1/2}$, x > 1 passing through the point $\left(2, \sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to:

[28-Jul-2022-Shift-2]

Options:

A. $11 + 6\log_e 3$

B. 19

C. $12 - 2\log_e 3$

D. $19 - 6\log_e 3$

Answer: B

Solution:

Solution:

$\frac{dy}{dx} + \frac{1}{x^2-1}y = \sqrt{\frac{x-1}{x+1}},\ x > 1$

Integrating factor I.F. $= e^{\int\frac{1}{x^2-1}dx} = e^{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|}$

$= \sqrt{\frac{x-1}{x+1}}$

Solution of differential equation

$y\sqrt{\frac{x-1}{x+1}} = \int\frac{x-1}{x+1}dx = \int\left(1-\frac{2}{x+1}\right)dx$

$y\sqrt{\frac{x-1}{x+1}} = x - 2\ln|x+1| + C$

Curve passes through $\left(2, \sqrt{\frac{1}{3}}\right)$

$\frac{1}{\sqrt{3}}\times\frac{1}{\sqrt{3}} = 2 - 2\ln 3 + C$

$C = 2\ln 3 - \frac{5}{3}$

$y(8)\times\frac{\sqrt{7}}{3} = 8 - 2\ln 9 + 2\ln 3 - \frac{5}{3}$

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Question74

The differential equation of the family of circles passing through the points (0, 2) and (0, −2) is :
[28-Jul-2022-Shift-2]

Options:

A. $2xy\frac{dy}{dx}+(x^2-y^2+4)=0$

B. $2xy\frac{dy}{dx}+(x^2+y^2-4)=0$

C. $2xy\frac{dy}{dx}+(y^2-x^2+4)=0$

D. $2xy\frac{dy}{dx}-(x^2-y^2+4)=0$

Answer: A

Solution:

Solution:
Family of circles passing through the points (0, 2) and (0, −2)
$x^2+(y-2)(y+2)+\lambda x=0,\ \lambda\in R$
$x^2+y^2+\lambda x-4=0\ldots\ldots\ (1)$
Differentiate w.r.t x
$2x+2y\frac{dy}{dx}+\lambda=0\ldots\ldots\ (2)$
Using (1) and (2), eliminate λ
$x^2+y^2-\left(2x+2y\frac{dy}{dx}\right)x-4=0$
$2xy\frac{dy}{dx}+x^2-y^2+4=0$

Question75

Let the solution curve $y=y(x)$ of the differential equation $(1+e^{2x})\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\ \frac{\pi}{2}\right)$. Then, $\lim_{x\to\infty}e^{x}y(x)$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{2}$

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Answer: B

Solution:

Solution:

D.E. $(1+e^{2x})\left(\frac{dy}{dx}+y\right)=1$

$\Rightarrow \frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

I.F. $=e^{\int 1 \cdot dx}=e^{x}$

$\therefore$ Solution

$e^{x}y(x)=\int \frac{e^{x}}{1+e^{2x}}dx$

$\Rightarrow e^{x}y(x)=\tan^{-1}(e^{x})+C$

$\because$ It passes through $\left(0,\frac{\pi}{2}\right)$, $C=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

$\therefore \lim\limits_{x\to\infty} e^{x}y(x)=\lim\limits_{x\to\infty}\tan^{-1}(e^{x})+\frac{\pi}{4}$

$=\frac{3\pi}{4}$

Question76

If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then [29-Jul-2022-Shift-2]

Options:

A. $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e(k^2+1)$

B. $\tan^{-1}\left(\frac{1}{k}\right)-\log_e(k^2+1)$

C. $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e(k^2+2k+2)$

D. $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

Answer: A

Solution:

Solution:

$\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

Let $x-1=X$, $y-1=Y$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$

Let $Y=tX \Rightarrow \frac{dY}{dX}=t+X\frac{dt}{dX}$

$t+X\frac{dt}{dX}=\frac{1+t}{1-t}$

$X\frac{dt}{dX}=\frac{1+t}{1-t}-t=\frac{1+t^2}{\ldots}$

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$\int \frac{1-t}{1+t^2} dt = \int \frac{dX}{X}$

$\tan^{-1} t - \frac{1}{2} \ln(1+t^2) = \ln|X| + c$

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1| + c$

Curve passes through (2, 1)

$0 - 0 = 0 + c \Rightarrow c = 0$

If $(k+1, 2)$ also satisfies the curve

$\tan^{-1}\left(\frac{1}{k}\right) - \frac{1}{2} \ln\left(\frac{1+k^2}{k^2}\right) = \ln k$

$2\tan^{-1}\left(\frac{1}{k}\right) = \ln(1+k^2)$

Question77

Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right) y = \frac{(x+3)}{x+1}$, $x > -1$, which passes through the point (0, 1). Then y(1) is equal to:
[29-Jul-2022-Shift-2]

Options:

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $\frac{7}{2}$

Answer: B

Solution:

Solution:

$\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right) y = \frac{(x+3)}{x+1}, x > -1$

Integrating factor I.F. $= e^{\int \frac{2x^2+11x+13}{x^3+6x^2+11x+6} dt}$

Let $\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$ A = 2, B = 1, C = −1

I.F. $= e^{(2\ln|x+1| + \ln|x+2| - \ln|x+3|)}$

$= \frac{(x+1)^2(x+2)}{x+3}$

Solution of differential equation

$y \cdot \frac{(x+1)^2(x+2)}{x+3} = \int (x+1)(x+2) dt$

$y \frac{(x+1)^2(x+2)}{x+3} = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$

Curve passes through (0, 1)

$1 \times \frac{1 \times 2}{3} = 0 + c \Rightarrow c = \frac{2}{3}$

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So, $y(1) = \dfrac{\frac{1}{3} + \frac{3}{2} + 2 + \frac{2}{3}}{\frac{(2^2 \times 3)}{4}} = \frac{3}{2}$

Question78

The population P = P(t) at time ' t ' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If P(0) = 850, then the time at which population becomes zero is :
24 Feb 2021 Shift 1

Options:

A. $\log_e 18$

B. $\log_e 9$

C. $1 / 2\log_e 18$

D. $2\log_e 18$

Answer: D

Solution:

Solution:

Given that $\frac{dP}{dt} = 0.5P - 450$

$\Rightarrow \int_0^t \frac{dp}{P - 900} = \int_0^t \frac{dt}{2}$

$\Rightarrow [\ell n \mid P(t) - 900 \mid]_0^t = \left[\frac{t}{2} \right]_0^t$

$\Rightarrow \ell n \mid P(t) - 900 \mid - \ell n \mid P(0) - 900 \Big| = \frac{t}{2}$

$\Rightarrow \ell n \mid P(t) - 900 \mid - \ell n \mid 50 \Big| = \frac{t}{2}$

For P(t) = 0

$\Rightarrow \ell n \left| \frac{900}{50} \right| = \frac{t}{2}$

$\Rightarrow t = 2\ell n 18$

Question79

Let f be a twice differentiable function defined on R, such that $f(0) = 1, f'(0) = 2$ and $f''(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval
[24 Feb 2021 Shift 2]

Options:

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A. (9, 12)

B. (6, 9)

C. (0, 3)

D. (3, 6)

Answer: B

Solution:

Given, $f(0) = 1$,
$f'(0) = 2$,
$f''(x) \neq 0$

$$\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$$

$\Rightarrow f(x)f''(x) - f'(x)f'(x) = 0$

$\Rightarrow \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$

$\Rightarrow \int \frac{f''(x)}{f'(x)} dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow \log f'(x) = \log f(x) + \log c$

$\Rightarrow f'(x) = cf(x)$

Now, put $x = 0$, we get

$f'(0) = cf(0)$

$\Rightarrow 2 = c \times 1$

$\Rightarrow c = 2$

Putting the value of $c = 2$ in Eq. (i), we get

$\log f'(x) = \log f(x) + \log 2$

$\Rightarrow f'(x) = 2f(x) \Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 2 dx$

$\Rightarrow \log f(x) = 2x + D \Rightarrow f(x) = e^{2x+D}$

$\Rightarrow f(x) = e^D \cdot e^{2x}$

$\Rightarrow f(x) = k \cdot e^{2x}$ [Let $k = e^D$]

Put $x = 0$, we get

$f(0) = k \cdot e^0$

$\Rightarrow 1 = k \Rightarrow f(x) = k \cdot e^{2x}$

$\therefore f(x) = e^{2x}$

Put $x = 1$, we get

$f(1) = e^2$

Clearly, e^2 lies in $(6, 9)$.

Question80

The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is .... [26 Feb 2021 Shift 1]

Answer: 2

Solution:

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Solution:

Given, $y^2 = a\left[x + \frac{\sqrt{a}}{2}\right]$, a > 0 ...(i)

Differentiating both sides w.r.t. ' x ',

$2y\frac{dy}{dx} = a[1+0] = a$ . . . (ii)

Use Eq. (ii) in Eq. (i) to eliminate the constant ' a '.

$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{2y}\sqrt{\frac{dy}{dx}}\right)$

$y^2 - 2xy\frac{dy}{dx} = 2\sqrt{2}\cdot y\sqrt{y}\cdot\frac{dy}{dx}\sqrt{\frac{dy}{dx}}$

Squaring on both sides,

$y^4 + 4x^2y^2\left(\frac{dy}{dx}\right)^2 - 4xy^3\frac{dy}{dx} = 8y^3\left(\frac{dy}{dx}\right)^3$

Thus, degree of above differential equation is 3 and its order is 1 .

Difference between degree and order $= 3 - 1 = 2$

Question81

Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is
[26 Feb 2021 Shift 2]

Options:

A. $\frac{18}{35}$

B. $-\frac{4}{3}$

C. $-\frac{18}{19}$

D. $-\frac{18}{11}$

Answer: C

Solution:

Solution:

Given, slope of tangent line to curve at (x, y) is $\frac{xy^2+y}{x}$

i.e., $\frac{dy}{dx} = \frac{xy^2+y}{x}$

$\Rightarrow \frac{dy}{dx} = y^2 + \frac{y}{x} \Rightarrow x\,dy = xy^2dx + y\,dx$

$\Rightarrow x\,dy - y\,dx = xy^2dx \Rightarrow \frac{x\,dy - y\,dx}{y^2} = x\,dx$

Integrating both sides, we get

$\frac{-x}{y} = \frac{x^2}{2} + C$ . . . (i)

The curve intersect line at $x = -2$

Then, $x = -2$, is satisfied by $x + 2y = 4$

Hence, $(-2) + 2y = 4$ Gives, $y = 3$

∴ Curve passes through $(2, -3)$.

Use (2, −3) is Eq. (i), we get

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$\frac{-2}{-3} = \frac{(-2)^2}{2} + C \Rightarrow C = \frac{-4}{3}$

∴ The curve is

$\frac{-x}{y} = \frac{x^2}{2} = \frac{-4}{3}$ . . . (ii)

It also passes through (3, y).

Put (3, 4) in Eq. (ii), we get

$\Rightarrow \frac{-3}{y} = \frac{(3)^2}{2} - \frac{4}{3} \Rightarrow y = -\frac{18}{19}$

Question82

Let $f(x) = \int_0^x e^t f(t) dt + e^x$ be a differentiable function for all $x \in R$. Then, $f(x)$ equals
[26 Feb 2021 Shift 2]

Options:

A. $2e^{(e^x - 1)} - 1$

B. $e^{e^x} - 1$

C. $2e^{e^x} - 1$

D. $e^{(e^x - 1)}$

Answer: A

Solution:

Solution:

Given, $f(x) = \int_0^x e^t f(t) dt + e^x$ . . . (i)

Since, $f(x)$ is differentiable function, differentiate Eq. (i)

$f'(x) = e^x f(x) + e^x$[Using Newton Leibnitz theorem]

$f'(x) = e^x(f(x) + 1) \Rightarrow \frac{f'(x)}{f(x) + 1} = e^x$

Integrating it, $\int \frac{f'(x)}{f(x) + 1} dx = \int e^x dx + C$

Let $f(x) + 1 = u$, then $f'(x) dx = du$

$\int \frac{du}{u} = e^x + C \Rightarrow \log u = e^x + C$

$\log(f(x) + 1) = e^x + C \quad [\because u = f(x) + 1] \ldots$ (ii)

Now

$f(x) = \int_0^x e^t f(t) dt + e^x \Rightarrow f(0) = e^0 = 1$

Put $x = 0$, in Eq. (ii), we get

$\log(2) = e^{\circ} + C \Rightarrow C = \log(2) - 1$

From Eq. (ii), we get

$\log(f(x) + 1) = e^x + \log 2 - 1$

$f(x) + 1 = e^{e^x + \log 2 - 1} = e^{e^x} \cdot e^{\log 2} \cdot e^{-1}$

$f(x) + 1 = 2e^{e^x} \cdot e^{-1} = 2e^{e^x - 1}$

$\therefore f(x) = 2e^{e^x - 1} - 1$

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The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2h. If the population of bacteria is 2000 after $\frac{k}{\log_e(6/5)}$h, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

[26 Feb 2021 Shift 1]

Options:

A. 4

B. 8

C. 2

D. 16

Answer: A

Solution:

Solution:
Let x be the number of bacteria at any time t.
Given that, $\frac{dx}{dt} \propto x$ ($\because$ Rate of growth $= \frac{dx}{dt}$)
$\Rightarrow \frac{dx}{dt} = \lambda x \Rightarrow \frac{dx}{x} = \lambda dt$
After integrating it, we get
$\log x = \lambda t + C \ldots$ (i)
$= 0, x = 1000$ which gives
$\log 1000 = 0 + C \Rightarrow C = \log 1000$
Given, when $t = 0$, $x =$ $\log 1000$
$\log x - \log 1000 = \lambda t$ or $\log\left(\frac{x}{1000}\right) = \lambda t \ldots$ (ii)
Given that in 2h, number of bacteria increased by 20% i.e. when $t = 2h$, $x = 1200$ Put, $t = 2$ and $x = 1200$ in Eq. (ii),
$\log\left(\frac{1200}{1000}\right) = 2\lambda$ gives, $\lambda = \frac{1}{2}\log\left(\frac{6}{5}\right)$
Again, from Eq. (ii),
$\log\left(\frac{x}{1000}\right) = \frac{1}{2}\log\left(\frac{6}{5}\right)$
or $\frac{x}{1000} = e^{\frac{t}{2}\log\left(\frac{6}{5}\right)} \ldots$ (iii)
Given, $x = 2000$ at $t = k/\log_e(6/5)$,
put in Eq. (iii),
$\frac{2000}{1000} = e^{\frac{k}{2}\log\left(\frac{6}{5}\right)/\log\left(\frac{6}{5}\right)}$
$2 = e^{k/2}$ or $\log 2 = k/2$
$k/\log 2 = 2$
$\therefore (k/\log_e 2)^2 = (2)^2 = 4$

Question84

If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$, y(0) = 0, then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is ....

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Answer: 1

Solution:

Solution:

Given $e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x\ ...\ (i)$

Let $e^{\sin y}=t$, then $e^{\sin y}\cdot\cos y\cdot\frac{dy}{dx}=\frac{dt}{dx}$

Putting in Eq. (i),

$\cos x$ ... (ii) (Linear form)

Then, IF $=e^{\int\cos x\,dx}=e^{\sin x}$

Solution of differential Eq. (ii) is,

$t\cdot IF=\int\cos x\cdot IF\,dx+C$

$t\cdot e^{\sin x}=\int\cos x\cdot e^{\sin x}dx+C$

$=e^{u}$ i.e. let $\sin x=u$ then $\cos x\,dx=du$

$\Rightarrow\ t\cdot e^{\sin x}=\int e^{u}du+C=e^{u}+C$

Put $u=\sin x$ and $t=e^{\sin y}$

$\Rightarrow\ e^{\sin y}\cdot e^{\sin x}=e^{\sin x}+C$

Given, $y(0)=0$, this gives $C=0$

$\Rightarrow\ e^{\sin y}\cdot e^{\sin x}=e^{\sin x}$

$\Rightarrow\ e^{\sin y+\sin x}=e^{\sin x}$

$\Rightarrow\ \sin y+\sin x=\sin x$

$\Rightarrow\ \sin y=0$

$\Rightarrow\ y=0$

$\therefore y(\pi/6)=y(\pi/3)=y(\pi/4)=0$

Hence, $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$

$=1+0+0+0=1$

Question85

If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point
[25 Feb 2021 Shift 1]

Options:

A. (5, 4)

B. (4, 5)

C. (4, 4)

D. (5, 5)

Answer: D

Solution:

Solution:

$x^2-4x+y+8$

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$\Rightarrow \frac{dy}{dx} = \frac{x^2 - 4x + y + 8}{x - 2} = \frac{(x-2)^2 + (y+4)}{(x-2)}$

$= (x-2) + \frac{y+4}{x-2}$

Let $(x-2) = t \Rightarrow dx = dt$

and $(y+4) = u \Rightarrow dy = du$

$\therefore \frac{dy}{dx} = \frac{du}{dt}$

Now, $\frac{dy}{dx} = (x-2) + \frac{(y+4)}{(x-2)}$

$\Rightarrow \frac{du}{dt} = t + \frac{u}{t} \Rightarrow \frac{du}{dt} - \frac{u}{t} = t$

Here, integrating factor (IF) $= 1/t$

$\Rightarrow u \cdot \left(\frac{1}{t}\right) = \int t\left(\frac{1}{t}\right)dt \Rightarrow u/t = t + c$

$\Rightarrow \frac{(y+4)}{(x-2)} = (x-2) + c$

$\because$ It passes through origin [i.e. (0, 0)], then

$\therefore \frac{4}{-2} = -2 + c$

$\Rightarrow -2 + 2 = c \Rightarrow c = 0$

Hence, $\frac{(y+4)}{(x-2)} = (x-2) + 0$

$\Rightarrow y + 4 = (x-2)^2$

Clearly, this curve passes through (5, 5) as it satisfies the equation.

Question86

If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines $2x - 3y = 1$ and$3x + 2y = 8$, then $|y(1)|$ is equal to ....
[25 Feb 2021 Shift 2]

Answer: 1

Solution:

Solution:

Given, $(2xy^2 - y)dx + xdy = 0$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -2y^2$

$\Rightarrow \frac{-1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = 2$ ...(i) [divide by y^2]

Let $\frac{1}{y} = v$, then $-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{dv}{dx}$, putting in Eq. (i)

$\frac{dv}{dx} + v\left(\frac{1}{x}\right) = 2$ (this is a linear form)

Now, integrating factor (IF) $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore (IF)v = \int 2.(IF)dx = \int 2xdx = 2\frac{x^2}{2} + C$

$\therefore (IA)v = x^2 + C$

Put $v = \frac{1}{y}$, this gives

$x^2 + c = \frac{x}{y}$

Now, first find point of intersection of lines

$\therefore$ The curve $x^2 + c = \frac{1}{y}$ passes through (2, 1).
Put $x = 2, y = 1$, we get $c = -2$
$\frac{x}{y} = x^2 - 2$
or $y = \frac{x}{x^2 - 2}$
Put $x = 1$, we get $y(1) = \frac{1}{1-2} = -1$
$\therefore |y(1)| = 1$

Question87

If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?
[24 Feb 2021 Shift 2]

Options:

A. 5

B. 10

C. $\frac{62}{5}$

D. $\frac{31}{5}$

Answer: B

Solution:

Solution:
Given, curve $y = f(x)$ passes through (1, 2) and satisfies
$x\frac{dy}{dx} + y = bx^4$
$\Rightarrow x\frac{dy}{dx} + y = bx^4$
$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = bx^3$
$\text{IF} = e^{\int \frac{1}{x} \cdot dx} = x$
$yx = \int bx^4 dx = \frac{bx^5}{5} + C$
$\therefore y = \frac{bx^4}{5} + \frac{C}{x} = f(x)$. . . (i)
$\because$ This curve passes through (1, 2).
$\therefore 2 \times 1 = \frac{b \times (1)^5}{5} + C$
$\Rightarrow 2 = \frac{b}{5} + C$ . . . (ii)
Also, $\int_1^2 f(x)dx = \frac{62}{5}$
$\Rightarrow \int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x}\right)dx = \frac{62}{5}$
$\Rightarrow \left[b \times \frac{x^5}{25} + C\log x\right]_1^2 = \frac{62}{5}$
$\Rightarrow \left[\left(\frac{b \times 32}{25} + C\log 2\right) - \left(\frac{b}{25} + c\log 1\right)\right] = \frac{62}{5}$

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[from Eq. (i)] On comparing, we get

$\frac{b}{25} \times 31 = \frac{62}{5}$ and $c = 0$

$b = \frac{62 \times 25}{31 \times 5}$

$b = 10$

Hence, the required value of $b = 10$.

Question88

The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is [18 Mar 2021 Shift 1]

Options:

A. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

B. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

C. $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

D. $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Answer: C

Solution:

Solution:

Given, equation of curve is $y^2 = 4a(x + a)$

$\Rightarrow y^2 = 4ax + 4a^2$...(i)

Differentiating Eq. (i) w.r.t. x, we get

$2y\frac{dy}{dx} = 4a$

$\Rightarrow a = \left(\frac{y}{2}\right) \cdot \frac{dy}{dx}$....(ii)

∴ Required differential equation is

$y^2 = 4 \times \frac{y}{2} \times \frac{dy}{dx} x + 4\left(\frac{y}{2} \cdot \frac{dy}{dx}\right)^2$ [From Eqs. (i) and (ii)]

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$

$\Rightarrow y\left[y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y\right] = 0$

As, $y \neq 0$

$\therefore y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

Question89

Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{(x^2 - y^2)}dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by

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10(α + β) is equal to
[18 Mar 2021 Shift 2]

Answer: 4

Solution:

Solution:

$x\,dy - y\,dx = \sqrt{x^2 - y^2}\,dx$

$\Rightarrow \dfrac{x\,dy - y\,dx}{x^2} = \dfrac{1}{x}\sqrt{1 - \dfrac{y^2}{x^2}}\,dx \Rightarrow \dfrac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \dfrac{1}{x}\,dx$

On integrating. $\int \dfrac{1}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} \cdot d\,(y/x) = \int \dfrac{1}{x}\,dx$

$\Rightarrow \sin^{-1}(y/x) = \log|x| + C$

Now, at $x = 1,\ y = 0$

$\therefore\ C = 0$

Hence, $y = x\sin(\log x)$

$\therefore\ A = \int_1^{e^x} x\sin(\log x)\,d\,x$

Put $x = e^t \Rightarrow dx = e^t\,dt$

$\therefore A = \int_0^{\pi} e^{2t}\sin(t)\,d\,t$

$\left(\text{using } \int e^{ax} \cdot \sin bx = \dfrac{e^{2x}}{a^2 + b^2}(a\sin bx - b\cos bx)\right)$

$\alpha e^{2\pi} + \beta = \left(\dfrac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \dfrac{1 + e^{2\pi}}{5}$

$\therefore\ \alpha = \dfrac{1}{5},\ \beta = \dfrac{1}{5}$

$\therefore\ 10(\alpha + \beta) = 4$

Question90

Which of the following is true for y(x) that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y$; y(0) = 0
[17 Mar 2021 Shift 1]

Options:

A. $y(1) = e^{-\frac{1}{2}} - 1$

B. $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

C. $y(1) = 1$

D. $y(1) = e^{\frac{1}{2}} - 1$

Answer: A

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Solution:

Solution:

$\frac{dy}{dx} = xy - 1 + x - y$

$\frac{dy}{dx} = x(y+1) - 1(y+1) \Rightarrow \frac{dy}{dx} = (x-1)(y+1)$

$\int \frac{dy}{y+1} = \int (x-1)dx$

$\log_e(y+1) = \frac{x^2}{2} - x + c \quad [\because \log_e 1 = 0]$

$\log_e 1 = 0 + C \Rightarrow C = 0$

$y(1) \Rightarrow \log_e(y+1) = \frac{1}{2} - 1$

$y + 1 = e^{-1/2}$

$y(1) = -1 + e^{-1/2}$

Question91

Let the curve y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(x+1)$.
If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to.........
[16 Mar 2021 Shift 1]

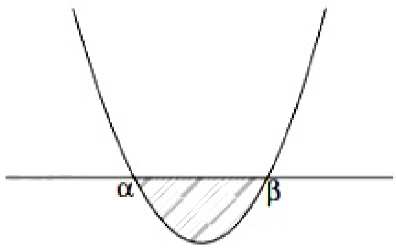
Answer: 2

Solution:

We have, $dy/dx = 2(x+1)$

$y = \int 2(x+1)dx = 2(x^2/2 + x) + C$

$y = x^2 + 2x + C$...(i)



Now, $\int_{\alpha}^{\beta} y\,dx = \frac{4\sqrt{8}}{3}$

$\int_{\alpha}^{\beta} (x^2 + 2x + c)dx = \frac{4\sqrt{8}}{3}$

$\Rightarrow \left[\frac{x^3}{3} + x^2 + cx\right]_{\alpha}^{\beta} = \frac{4\sqrt{8}}{3}$

$\Rightarrow \left(\frac{\beta^3 - \alpha^3}{3}\right) + (\beta^2 - \alpha^2) + c(\beta - \alpha) = \frac{4\sqrt{8}}{3}$....(ii)

From Eq. (i),

$\alpha + \beta = -2$

$\alpha\beta = c$

$\therefore \beta - \alpha = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4c} = 2\sqrt{1-c}$

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From Eq. (ii), $\frac{1}{3}(\beta-\alpha)(\alpha^2+\beta^2+\alpha\beta)+(\beta-\alpha)(\beta+\alpha)+c(\beta-\alpha)=\frac{4\sqrt{8}}{3}$

$\Rightarrow \frac{1}{3}(2\sqrt{1-c})(4-c)+(2\sqrt{1-c})(-2+c)=\frac{4\sqrt{8}}{3}$

$\Rightarrow (2\sqrt{1-c})[4-c+(-6+3c)]=4\sqrt{8}$

$\Rightarrow (2\sqrt{1-c})[2(c-1)]=4\sqrt{8}$

$\Rightarrow (1-c)^{3/2}=-\sqrt{8}$

$\Rightarrow (1-c)^3=8$

$\Rightarrow 1-c=2$

$\therefore c=-1$

Now, $y=x^2+2x-1$

$\therefore y(1)=1^2+2\cdot1-1=2$

Question92

Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2, x>0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves C_1 and C_2 is equal to

[16 Mar 2021 Shift 2]

Options:

A. $\pi-1$

B. $\frac{\pi}{2}-1$

C. $\pi+1$

D. $\frac{\pi}{4}+1$

Answer: B

Solution:

Solution:

Given, $2xy\frac{dy}{dx}=y^2-x^2, x>0$

$\frac{dy}{dx}=\left(\frac{y^2-x^2}{2xy}\right), x>0$

Let $y=vx$

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2x\cdot vx}=\frac{v^2-1}{2v}$

$\Rightarrow x\frac{dv}{dx}=\frac{v^2-1}{2v}-v\Rightarrow x\frac{dv}{dx}=\frac{v^2-1-2v^2}{2v}$

$\Rightarrow x\frac{dv}{dx}=\frac{-(v^2+1)}{2v}\Rightarrow\int-\left(\frac{2v}{v^2+1}\right)dv=\int\frac{dx}{x}$

$\Rightarrow -\log|v^2+1|=\log x+c$

$\Rightarrow \log|x|+\log|v^2+1|=c$

$\Rightarrow \log|v^2+1)x|=c\Rightarrow\left(\frac{y^2}{x^2}+1\right)x=c$

$\Rightarrow \frac{(y^2+x^2)}{x}=c\Rightarrow y^2+x^2=cx$

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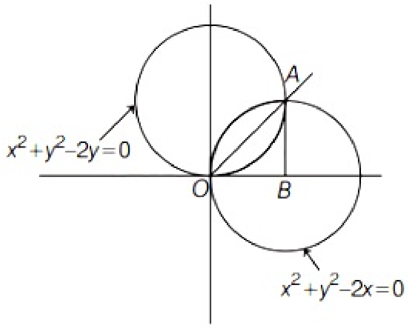
$C_1 \Rightarrow 1 + 1 = C_1$
$\Rightarrow C_1 = 2$
$C_1 \Rightarrow x^2 + y^2 = 2x$
$C_2 \Rightarrow x^2 + y^2 = C_2 y$
$\Rightarrow 1 + 1 = C_2 \Rightarrow C_2 = 2$
$C_2 \Rightarrow x^2 + y^2 = 2y$



$\therefore$ Required area $= 2\left[\frac{\pi \cdot 1^2}{4} - \frac{1}{2} \cdot 1 \cdot 1\right]$
$= 2\left[\left(\frac{\pi - 2}{4}\right)\right] = \frac{\pi}{2} - 1$

Question93

Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y + 1)\left[(y + 1)e^{x^2/2} - x\right]$ $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\frac{-e^{3/2}}{(e^2 + 1)^2}$

B. $-\frac{2e^2}{(1 + e^2)^2}$

C. $\frac{e^{5/2}}{(1 + e^2)^2}$

D. $\frac{5e^{1/2}}{(e^2 + 1)^2}$

Answer: A

Solution:

Solution:
Given, $\frac{dy}{dx} = (y + 1)\left[(y + 1)e^{x^2/2} - x\right]$...(i)
$\Rightarrow \frac{dy}{dx} = (y + 1)^2 e^{x^2/2} - x(y + 1)$
$\Rightarrow \frac{dy}{dx} + x(y + 1) = (y + 1)^2 e^{x^2/2}$
$\Rightarrow \frac{1}{(y + 1)^2} \cdot \frac{dy}{dx} + \frac{1}{y + 1} \cdot x = e^{x^2/2}$...(ii)
Let $\frac{1}{y + 1} = t \Rightarrow \frac{-1}{(y + 1)^2} \cdot \frac{dy}{dx} = \frac{dt}{dx}$

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$\Rightarrow \; -\frac{dt}{dx} + tx = e^{x^2/2}$

$\Rightarrow \; \frac{dt}{dx} + (-x)t = -e^{x^2/2}$

which is linear differential equation

$IF = e^{\int -x\,dx} = e^{-x^2/2}$

Now, solution of differential equation is,

$t \cdot e^{-x^2/2} = -\int e^{-x^2/2} \cdot e^{x^2/2} dx$

$\Rightarrow \left(\frac{1}{y+1}\right) e^{-x^2/2} = -x + c$, where c is constant of integration....(iii)

Given, $y(2) = 0$ i.e. when $x = 2$, then $y = 0$.

From Eq. (iii),

$e^{-2} = -2 + c \Rightarrow c = 2 + e^{-2}$

Now, at $x = 1$

$\frac{1}{y+1} e^{-1/2} = -1 + e^{-2} + 2$

$\Rightarrow \; (y+1) = \frac{e^{-1/2}}{1+e^{-2}}$

Now, putting the value of $(y + 1)$ in Eq. (i), we get

$y'(1) = \frac{e^{-1/2}}{1+e^{-2}}\left(\frac{e^{-1/2}}{1+e^{-2}} \cdot e^{1/2} - 1\right)$

$= \frac{e^{-1/2}}{1+e^{-2}}\left(\frac{1}{1+e^{-2}} - 1\right)$

$= \frac{e^{-1/2}}{1+e^{-2}}\left(\frac{1-1-e^{-2}}{1+e^{-2}}\right)$

$= \frac{-e^{-5/2}}{(1+e^{-2})^2} = \frac{-e^{-5/2}}{\frac{(e^2+1)^2}{e^4}} = \frac{-e^{3/2}}{(e^2+1)^2}$

Question94

Let $y = y(x)$ be the solution of the differential equation $\cos x(3\sin x + \cos x + 3)\,dy = [1 + y\sin x(3\sin x + \cos x + 3)]dx$

$0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to

[17 Mar 2021 Shift 2]

Options:

A. $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$

B. $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

C. $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$

D. $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

Answer: B

Solution:

Solution:

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$\Rightarrow \cos x\, dy = \left(\frac{1}{3\sin x + \cos x + 3} + y\sin x\right)dx$

$\Rightarrow \cos x \cdot \frac{dy}{dx} = y\sin x + \frac{1}{3\sin x + \cos x + 3}$

$\Rightarrow \frac{dy}{dx} = y\frac{\sin x}{\cos x} + \frac{1}{\cos x(3\sin x + \cos x + 3)}$

$\Rightarrow \frac{dy}{dx} - (\tan x)y = \frac{1}{\cos x(3\sin x + \cos x + 3)}$....(i)

Which is linear differential equation.

$\therefore$ Integrating factor $(IF) = e^{\int(-\tan x)dx}$

$= e^{\log|\cos x|} = |\cos X|$

$\because |\cos x| > 0, \forall x \in [0, \pi/2)$

$\therefore |\cos x| = \cos x$

Hence, solution of Eq. (i) is

$y(\cos x) = \int(\cos x)\cdot\frac{1}{\cos x(3\sin x + \cos x + 3)}dx$

$\Rightarrow y\cos x = \int\frac{1}{3\sin x + \cos x + 3}dx$

$\Rightarrow y\cos x = \int\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2} + 6\tan\frac{x}{2} + 4}dx = \int\frac{\sec^2\frac{x}{2}dx}{2\left(\tan^2\frac{x}{2} + 3\tan\frac{x}{2} + 2\right)}dx$

Putting $\tan\frac{x}{2} = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

$\therefore y\cos x = \int\frac{dz}{z^2 + 3z + 2} = \int\frac{dz}{(z+1)(z+2)}$

$= \int\frac{1}{z+1}dz - \int\frac{1}{z+2}dz = \log(z+1) - \log(z+2) + c$

$\Rightarrow y\cos x = \log\left|\frac{z+1}{z+2}\right| + c = \log\left|\frac{\tan\frac{x}{2} + 1}{\tan\frac{x}{2} + 2}\right| + c$...(i)

Since, $y(0) = 0$

$\therefore\ C = -\log\left(\frac{1}{2}\right) = \log 2$

From Eq. (i), $y\cos x = \log\left|\frac{\tan\frac{x}{2} + 1}{\tan\frac{x}{2} + 2}\right| + \log 2$

$\therefore\ y\left(\frac{\pi}{3}\right) = 2\left[\log\left|\frac{\frac{1}{\sqrt{3}} + 1}{\frac{1}{\sqrt{3}} + 2}\right| + \log 2\right] = 2\log\left|2\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1}\right)\right|$

$= 2\log\left|2\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}\right)\right| = 2\log\left|\frac{2\sqrt{3}+10}{11}\right|$

Question95

If the curve $y = y(x)$ is the solution of the differential equation $2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}dx$, $x > 0$ which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of $y(16)$ is equal to

[17 Mar 2021 Shift 2]

Options:

A. $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

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C. $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

D. $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

Answer: C

Solution:

Solution:
Given, $2(x^2+x^{5/4})dy - y\cdot(x+x^{1/4})dx =$
$2x^{9/4}dx$, where $x>0$
After rearranging, we get
$$\frac{dy}{dx}-\frac{y}{2x}=\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$$
This is of the form $\frac{dy}{dx}+Py=Q$, where P and Q are constants or function of x.
∴ Integrating factor (IF) $=e^{\int Pdx}$
$$=e^{\int -\frac{1}{2x}dx}=e^{\frac{-1}{2}\int x dx}$$
$$=e^{-\frac{1}{2}\log x}=e^{\log(x)^{-1/2}}=\frac{1}{x^{1/2}}$$
Its solution is
$y\times(IF)=\int Q\times(IF)dx$
$$\Rightarrow y\times\frac{1}{x^{1/2}}=\int\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}\times x^{-1/2}dx=\int\frac{(x)^{\frac{9}{4}-\frac{5}{4}-\frac{1}{2}}}{(x^{3/4}+1)}dx$$
$$\Rightarrow y\times\frac{1}{x^{1/2}}=\int\frac{x^{1/2}dx}{(x^{3/4}+1)}\ldots(i)$$
Putting $x=z^4$
$\Rightarrow dx=4z^3\cdot dz$
RHS of Eq. (i) becomes,
$$\int\frac{z^2\cdot 4z^3}{(z^3+1)}\cdot dz=4\int\frac{z^2(z^3+1-1)}{(z^3+1)}dz$$
$$=4\left[\int\frac{z^2(z^3+1)}{(z^3+1)}dz-\int\frac{z^2}{(z^3+1)}dz\right]$$
$$=4\left[\frac{z^3}{3}-\frac{1}{3}\cdot\int\frac{3z^2}{z^3+1}dz\right]$$
$$=4\left[\frac{z^3}{3}-\frac{1}{3}\cdot\log|z^3+1|\right]$$
$$\left(\because\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+C\right)$$
$$=\frac{4z^3}{3}-\frac{4}{3}\log|z^3+1|+C$$
$$=\frac{4x^{3/4}}{3}-\frac{4}{3}\log|x^{3/4}+1|+C\quad\left(\begin{array}{c}\because x=z^4\\ \therefore x^{3/4}=z^3\end{array}\right)$$
∴ Eq. (i) becomes,
$$y\times x^{-1/2}=\frac{4x^{3/4}}{3}-\frac{4}{3}\log|x^{3/4}+1|+C$$
Since, this passes through $\left(1, 1-\frac{4}{3}\log_e 2\right)$
Then,
$$\left(1-\frac{4}{3}\log_e 2\right)\times 1=\frac{4\times 1}{3}-\frac{4}{3}\log\left|1+1\right|+C$$
$$\Rightarrow C=1-\frac{4}{3}\Rightarrow C=\frac{-1}{3}$$
Hence,
$$y=\frac{4}{3}x^{5/4}-\frac{4}{3}\sqrt{x}\log x^{3/4}+1\Big|-\frac{\sqrt{x}}{3}$$
$\because x>0$

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$\therefore\ y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\log(x^{3/4} + 1) - \frac{\sqrt{x}}{3}$
Now, putting x = 16, we get
$y(16) = \frac{4}{3} \times 32 - \frac{4}{3} \times 4\log 9 - \frac{4}{3}$
$= \frac{124}{3} - \frac{32}{3}\log 3$
$= 4\left(\frac{31}{3} - \frac{8}{3}\log 3\right)$

Question96

If y = y(x) is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x,\ y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function y(x) over R is equal to [16 Mar 2021 Shift 1]

Options:

A. 8

B. $\frac{1}{2}$

C. $-\frac{15}{4}$

D. $\frac{1}{8}$

Answer: D

Solution:

Solution:
Given, $\frac{dy}{dx} + 2y\tan x = \sin x$
This differential equation is of the form $\frac{dy}{dx} + Py = Q$ where P and Q is function of x.
which is a linear differential equation.
Here, $P = 2\tan x$ and $Q = \sin x$
The integrating factor of linear differential equation is $e^{\int P dx}$.
Here, $e^{\int 2\tan x\, dx} = e^{\int \frac{2\sin x}{\cos x}dx} = e^{-2\log(\cos x)} = \sec^2 x$
Now, $\frac{dy}{dx} + 2y\tan x = \sin x$
On multiplying $\sec^2 \times$ both the sides,
$\sec^2 x\frac{dy}{dx} + 2y\sec^2 x\tan x = \sin x\sec^2 x$
$\Rightarrow\ \frac{d}{dx}(y\sec^2 x) = \sin x\sec^2 x$
$\Rightarrow\ y\sec^2 x = \int \sin x\sec^2 x\, dx$
$\Rightarrow\ y\sec^2 x = \int \frac{\sin x}{\cos^2 x}dx$
Let $\cos x = t$
$(-\sin x)dx = dt$
$\int \frac{-dt}{t^2} = \frac{1}{t} + c$
So, $y\sec^2 x = \frac{1}{\cos x} + c$

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Now, $x = \pi / 3, y = 0$
$0 = 2 + c$
$\Rightarrow c = -2$
So, $y\sec^2 x = \sec x - 2$

$$y = \cos^2 x\left(\frac{1}{\cos x} - 2\right) = -2\cos^2 x + \cos x$$

$$\Rightarrow y = -2\left(\cos^2 x - \frac{\cos x}{2}\right)$$

$$\Rightarrow y = -2\left[\left(\cos^2 x - \frac{\cos x}{2} + \frac{1}{16}\right) - \frac{1}{16}\right]$$

$$\Rightarrow y = -2\left[(\cos x - 1/4)^2 - \frac{1}{16}\right]$$

$$\Rightarrow y = \frac{1}{8} - 2(\cos x - 1/4)^2$$

So, $y_{min} = \frac{1}{8}$

Question97

If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)y = \sin x, 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to [16 Mar 2021 Shift 2]

Options:

A. $\frac{1}{4}\log_e 2$

B. $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

C. $\log_e 2$

D. $\frac{1}{2}\log_e 2$

Answer: B

Solution:

Solution:

Given, $\frac{dy}{dx} + (\tan x)y = \sin x, x \in \left[0, \frac{\pi}{3}\right]$

which is a linear differential equation of the form of $\frac{dy}{dx} + Py = Q$

Here, $P = \tan x$

$\therefore \quad IF = e^{\int P dx}$

$\Rightarrow e^{\int \tan x dx} = e^{\log(\sec x)} = \sec x$

Multiplying by $\sec x$ on both sides

$$\frac{dy}{dx} + (\tan x)y = \sin x$$

$$\sec x\frac{dy}{dx} + (\tan x \sec x)y = \sin x \sec x$$

$$\Rightarrow \frac{d}{dx}(y\sec x) = \tan x \Rightarrow y\sec x = \int \tan x\, dx$$

$\Rightarrow \quad y\sec x = \log(\sec x) + c$

$y = \cos x \log(\sec x) + c \cdot \cos x$

$y(0) = 0$

$\Rightarrow 0 = 1 \cdot 0 + c \cdot 1 \Rightarrow c = 0$

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$\Rightarrow y\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) \cdot \log\left(\sec\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}\log(\sqrt{2})$

$= \frac{1}{2\sqrt{2}}\log 2$

Question98

In a triangle ABC, if $\left|\overrightarrow{BC}\right| = 8, \left|\overrightarrow{CA}\right| = 7, \left|\overrightarrow{AB}\right| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\frac{25}{4}$

B. $\frac{85}{14}$

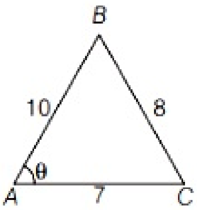
C. $\frac{127}{20}$

D. $\frac{115}{16}$

Answer: B

Solution:

Projection of AB or AC = AB cos theta = 10 cdot cos theta



$= 10 . \left(\frac{10^2 + 7^2 - 8^2}{2 \times 7 \times 10}\right) = \frac{85}{14} \left(\text{using } \cos\theta = \frac{c^2 + b^2 - a^2}{2bc}\right)$

Question99

Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to:
[25 Jul 2021 Shift 1]

Options:

A. $(2 - \sqrt{3}) - \log_e 2$

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C. $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$

D. $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$

Answer: D

Solution:

Solution:

$\frac{dy-dx}{e^{y-x}} = x\,dx$

$\Rightarrow \frac{dy-dx}{e^{y-x}} = x\,dx$

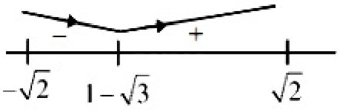
$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$

At $x = 0, y = 0 \Rightarrow c = -1$

$\Rightarrow e^{x-y} = \frac{2-x^2}{2}$

$\Rightarrow y = x - \ln\left(\frac{2-x^2}{2}\right)$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2-x^2} = \frac{2+2x-x^2}{2-x^2}$



So minimum value occurs at $x = 1-\sqrt{3}$

$y(1-\sqrt{3}) = (1-\sqrt{3}) - \ln\left(\frac{2-(4-2\sqrt{3})}{2}\right)$

$= (1-\sqrt{3}) - \ln(\sqrt{3}-1)$

Question100

Let $y = y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0) = 0$ If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha\log_e 2$, then the value of α is equal to: [27 Jul 2021 Shift 1]

Options:

A. $-\frac{1}{4}$

B. $\frac{1}{4}$

C. 2

D. $-\frac{1}{2}$

Answer: A

Solution:

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$\frac{dy}{dx} = e^{3x}.e^{4y} \Rightarrow \int e^{-4y}dy = \int e^{3x}dx$

$\frac{e^{-4y}}{-4} = e^{3x}3 + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x} - 7}{-3}$

$e^{4y} = \frac{3}{7 - 4e^{3x}} \Rightarrow 4y = \ln\left(\frac{3}{7 - 4e^{3x}}\right)$

$4y = \ln\left(\frac{3}{6}\right)$ when $x = -\frac{2}{3}\ln 2$

$y = \frac{1}{4}\ln\left(\frac{1}{2}\right) = -\frac{1}{4}\ln 2$

Question101

If $y = y(x), y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation $\sec y\frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with $y(0) = 0$ then $5y'\left(\frac{\pi}{2}\right)$ is equal to ________.
[27 Jul 2021 Shift 1]

Answer: 2

Solution:

Solution:

$\sec y\frac{dy}{dx} = 2\sin x\cos y$

$\sec^2 y dy = 2\sin x dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$\sec^2 y\frac{dy}{dx} = 2\sin x$

$5\frac{dy}{dx} = 2$

Question102

Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx, x > 2$If $y(3) = 3$, then $y(4)$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 4

B. 12

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C. 8

D. 16

Answer: B

Solution:

Solution:

$(x - x^3)d y = (y + yx^2 - 3x^4)d x$

$\Rightarrow xd y - yd x = (yx^2 - 3x^4)d x + x^3 d y$

$\Rightarrow \frac{xd y - yd x}{x^2} = (yd x + xd y) - 3x^2 d x$

$\Rightarrow d\left(\frac{y}{x}\right) = d(xy) - d(x^3)$

Integrate $\Rightarrow \frac{y}{x} = xy - x^3 + c$

given $f(3) = 3$

$\Rightarrow \frac{3}{3} = 3 \times 3 - 3^3 + c$

$\Rightarrow c = 19$

$\therefore \frac{y}{x} = xy - x^3 + 19$

at $x = 4$, $\frac{y}{4} = 4y - 64 + 19$

$15y = 4 \times 45$

$\Rightarrow y = 12$

Question103

Let $y = y(x)$ be the solution of the differential equation $d y = e^{\alpha x + y} d x$; $\alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to ____.
[27 Jul 2021 Shift 2]

Answer: 2

Solution:

$\int e^{-y} d y = \int e^{ex} d x$

$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c$ ....(i)

Put $(x, y) = (l n2, \ln 2)$

$\frac{-1}{2} = \frac{2^{\alpha}}{\alpha} + C$ ....(ii)

Put $(x, y) \equiv (0, -\ln 2)$ in (i)

$-2 = \frac{1}{\alpha} + C$ ......(iii)

(ii) - (iii)

$\frac{2^{\alpha} - 1}{\alpha} = \frac{3}{2}$

$\Rightarrow \alpha = 2$ (as $\alpha \in N$)

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Question104

Let $y = y(x)$ be the solution of the differential equation $x dy = (y + x^3 \cos x) dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:
[25 Jul 2021 Shift 2]

Options:

A. $\frac{\pi^2}{4} + \frac{\pi}{2}$

B. $\frac{\pi^2}{2} + \frac{\pi}{4}$

C. $\frac{\pi^2}{2} - \frac{\pi}{4}$

D. $\frac{\pi^2}{4} - \frac{\pi}{2}$

Answer: A

Solution:

Solution:

$x dy = (y + x^3 \cos x) dx$

$x dy = y dx + x^3 \cos x dx$

$\frac{x dy - y dx}{x^2} = \frac{x^3 \cos x dx}{x^2}$

$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x dx$

$\Rightarrow \frac{y}{x} = x \sin x - \int 1 . \sin x dx$

$\frac{y}{x} = x \sin x + \cos x + C$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$

so $\frac{y}{x} = x \sin x + \cos x + 1$

$y = x^2 \sin x + x \cos x + x \quad x = \frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

Question105

Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to _______.
[25 Jul 2021 Shift 2]

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Answer: 1

Solution:

Solution:

$y' = \frac{2y}{x \ln x}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x \ln x}$

$\Rightarrow \ln|y| = 2\ln|\ln x| + C$

put $x = 2$, $y = (\ln 2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ln x)^2$

$\Rightarrow f(e) = 1$

Question106

Let $y = y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy, y(1) = 1$. If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to ______.
[22 Jul 2021 Shift 2]

Answer: 4

Solution:

Solution:

$y + 1 = Y \Rightarrow dy = dY$

$x + 2 = X \Rightarrow dx = dX$

$\Rightarrow \left(X e^{\frac{Y}{X}} + Y\right) dX = X dY$

$\Rightarrow X dY - Y dX = X e^{Y/x} dX$

$\Rightarrow d\left(\frac{Y}{X}\right) e^{-\frac{Y}{X}} = \frac{dX}{X}$

$-e^{-Y/x} = l|X| + c$

$(3, 2) \rightarrow -e^{-\frac{2}{3}} = l|3| + c$

$-e^{-\frac{Y}{X}} = \ln|X| - e^{-\frac{2}{3}} - \ln 3$

$e^{-\frac{Y}{X}} = e^{2/3} + \ln 3 - \ln|X| > 0$

$\ln|X| < (e^{2/3} + \ln 3)$

Let $\lambda = (e^{2/3} + \ln 3)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$-e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although $x = -2$ should be excluded from domain but according to the given problem it will the most appropriate solution.

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Question107

Let y = y(x) be the solution of the differential equation $x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx - 1 \le x \le 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves x = 0, $x = \frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is:
[20 Jul 2021 Shift 1]

Options:

A. $\frac{1}{8}(\pi - 1)$

B. $\frac{1}{12}(\pi - 3)$

C. $\frac{1}{4}(\pi - 2)$

D. $\frac{1}{6}(\pi - 1)$

Answer: A

Solution:

Solution:
We have

$\frac{dy}{dx} = \frac{x\left(\frac{y}{x}.\tan\frac{y}{x} - 1\right)}{x\tan\frac{y}{x}}$

$\therefore \frac{dy}{dx} = \frac{y}{x} - \cot\left(\frac{y}{x}\right)$

Put $\frac{y}{x} = v$

$\Rightarrow y = vn$

$\therefore \frac{dy}{dx} = v + \frac{nd v}{dx}$

Now, we get $v + n\frac{dv}{dx} = v - \cot(v)$

$\Rightarrow \int(\tan)dv = -\int\frac{dx}{x}$

$\therefore \ln\left|\sec\left(\frac{y}{x}\right)\right| = -\ln|x| + c$

As $\left(\frac{1}{2}\right) = \left(\frac{y}{x}\right) \Rightarrow C = 0$

$\therefore \sec\left(\frac{y}{x}\right) = \frac{1}{x}$

$\Rightarrow \cos\left(\frac{y}{x}\right) = x$

$\therefore y = x\cos^{-1}(x)$

So, required bounded area

$= \int_0^{1/\sqrt{2}} x(\cos^{-1}x)dx = \left(\frac{\pi - 1}{8}\right)$

$\therefore$ option (1) is correct.

Question108

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Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0$, $y(1) = -1$. Then the value of $(y(3))^2$ is equal to:
[20 Jul 2021 Shift 1]

Options:

A. $1 - 4e^3$

B. $1 - 4e^6$

C. $1 + 4e^3$

D. $1 + 4e^6$

Answer: B

Solution:

Solution:

$e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

$\Rightarrow e^x\sqrt{1-y^2}\,dx + \frac{-y}{x}dy$

$\Rightarrow \int \frac{-y}{\sqrt{1-y^2}}dy = \int e^x x\,dx$

$\Rightarrow \sqrt{1-y^2} = e^x(x-1) + c$

Given : At $x = 1, y = -1$

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\therefore \sqrt{1-y^2} = e^x(x-1)$

At $x = 3$ $1 - y^2 = (e^3 2)^2 \Rightarrow y^2 = 1 - 4e^6$

Question109

Let a curve y = y(x) be given by the solution of the differential equation
$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}\,dy$$
If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^α is equal to _______.
[20 Jul 2021 Shift 2]

Answer: 2

Solution:

Solution:

$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}\,dy$

Put $\cos^{-1}(e^{-x})\theta$, $\theta \in [0, \pi]$

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$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^{x}+1}{2c^{x}}}$

$\sqrt{\frac{e^{x}+1}{2c^{x}}}dx = \sqrt{e^{2x}-1}dy$

$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

Put $e^{x} = t, \frac{dt}{dx} = e^{x}$

$\frac{1}{\sqrt{2}}\int\frac{dt}{e^{x}\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

$\int\frac{dt}{t\sqrt{t^{2}-t}} = \sqrt{2}y$

Put $t = \frac{1}{z}, \frac{dt}{dz} = -\frac{1}{z^{2}}$

$\int\frac{-\frac{dz}{z^{2}}}{\frac{1}{z}\sqrt{\frac{1}{z^{2}}-\frac{1}{z}}} = \frac{\sqrt{2}}{y}$

$-\int\frac{dz}{\sqrt{1-z}} = \sqrt{2}y$

$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2(1-e^{-x})^{1/2} = \sqrt{2}y + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1)$, passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-e^{-\alpha} = \frac{1}{2}$

$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$

Question110

Let $y = y(x)$ be solution of the following differential equation $e^{y}\frac{dy}{dx} - 2e^{y}\sin x + \sin x\cos^{2}x = 0$, $y\left(\frac{\pi}{2}\right) = 0$ If $y(0) = \log_{e}(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to _______.
[25 Jul 2021 Shift 1]

Answer: 4

Solution:

Solution:

Let $e^{y} = t$

$\Rightarrow\frac{dt}{dx} - (2\sin x)t = -\sin x\cos^{2}x$

I.F. $= e^{2\cos x}$

$\Rightarrow t \cdot e^{2\cos x} = \int e^{2\cos x} \cdot (-\sin x\cos^{2}x)dx$

$\Rightarrow e^{y} \cdot e^{2\cos x} = \int e^{2z} \cdot z^{2}dz, z = e^{2\cos x}$

at $x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{}$

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$\Rightarrow e^y = \frac{1}{2}\cos^2 x - \frac{1}{2}\cos x + \frac{1}{4} + \frac{3}{4} \cdot e^{-2\cos x}$

$\Rightarrow y = \log\left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4}e^{-2\cos x}\right]$

Put $x = 0$

$\Rightarrow y = \log\left[\frac{1}{4} + \frac{3}{4}e^{-2}\right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$

Question111

Let $y = y(x)$ be the solution of the differential equation $\operatorname{cosec}^2 x\,dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\,dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to:
[22 Jul 2021 Shift 2]

Options:

A. $e^{1/2}$

B. $e^{-1/2}$

C. e^{-1}

D. e

Answer: C

Solution:

Solution:

$\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$

$\Rightarrow \frac{dy}{dx} + (-\cos 2x)y = \cos 2x$

I.F. $= e^{\int -\cos 2x\,dx} = e^{-\frac{\sin 2x}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx + c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$

Given

$y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 0 = -e^{-\frac{1}{2}} + c \Rightarrow c = e^{-\frac{1}{2}}$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$

at $x = 0$

$y = -1 + e^{-\frac{1}{2}}$

$\Rightarrow y(0) = -1 + e^{-\frac{1}{2}} \Rightarrow (y(0) + 1)^2 = e^{-1}$

Question112

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Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$,for all $x > 0$, where

$A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

[20 Jul 2021 Shift 2]

Options:

A. $\frac{\pi}{2} + \frac{4}{\pi}$

B. $\frac{\pi}{2} - \frac{1}{\pi}$

C. $\frac{3\pi}{2} - \frac{1}{\pi}$

D. $\frac{\pi}{2} - \frac{4}{\pi}$

Answer: A

Solution:

Solution:

$|A| = -\frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} = |A|$

$\frac{dy}{dx} = -\frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} + \frac{y}{x} = 2\sin x + 2$

I.F. $= e^{\int \frac{1}{x} dx} = x$

$\Rightarrow yx = \int x(2\sin x + 2)dx$

$xy = x^2 - 2x\cos x + 2\sin x + c$ ....(i)

Now $x = \pi$, $y = \pi + 2$

Use in (i) $c = 0$

Now (i) be comes

$xy = x^2 - 2x\cos x + 2\sin x$

put $x = \pi / 2$

$\frac{\pi}{2}y = \left(\frac{\pi}{2}\right)^2 - 2 . \frac{\pi}{2}\cos\frac{\pi}{2} + 2\sin\frac{\pi}{2}$

$\frac{\pi}{2}y = \frac{\pi^2}{4} + 2$

Question113

Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers.

If $\vec{a} . \vec{b} = -1$ and $\vec{b} . \vec{c} = 10$, then $\left(\vec{a} \times \vec{b}\right) . \vec{c}$ is equal to ____ .

[27 Jul 2021 Shift 2]

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Answer: 9

Solution:

Solution:
$\vec{a} = (1, -\alpha, \beta)$
$\vec{b} = (3, \beta, -\alpha)$
$\vec{c} = (-\alpha, -2, 1)$; $\alpha, \beta \in I$
$\vec{a} . \vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1$
$\Rightarrow \alpha\beta = 2$

1	2
2	1
−1	−2
−2	−1

$\vec{b} . \vec{c} = 10$
$\Rightarrow -3\alpha - 2\beta - \alpha = 10$
$\Rightarrow 2\alpha + \beta + 5 = 0$
$\therefore \alpha = -2;\ \beta = -1$

$$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$
$= 1(-1 + 4) - 2(3 - 4) - 1(-6 + 2)$
$= 3 + 2 + 4 = 9$

Question114

A differential equation representing the family of parabolas with axis parallel to Y-axis and whose length of latus rectum is the distance of the point (2, −3) from the line 3x + 4y = 5, is given by
[27 Aug 2021 Shift 2]

Options:

A. $10\frac{d^2y}{dx^2} = 11$

B. $11\frac{d^2x}{dy^2} + 10$

C. $10\frac{d^2x}{dy^2} = 11$

D. $11\frac{d^2y}{dx^2} = 10$

Answer: D

Solution:

Let (h, k) be the vertex of parabola. Then, equation of parabola parallel to Y-axis is
$(x - h)^2 = 4a(y - k)$ ...(i)
Also,

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Length of latusrectum = Distance of point (2, − 3) from the line
$3x + 4y = 5$
$\Rightarrow 4a = \frac{|6 - 12 - 5|}{\sqrt{3^2 + 4^2}}$
$\Rightarrow 4a = \frac{11}{5}$
∴ From Eq. (i),
$(x - h)^2 = \frac{11}{5}(y - k)$
Differentiating w.r.t. x, we get
$2(x - h) = \frac{11}{5}\frac{dy}{dx}$
Again, differentiating w.r.t. x
$2 = \frac{11}{5}\frac{d^2y}{dx^2}$
$\Rightarrow 11\frac{d^2y}{dx^2} = 10$

Question115

Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1 - (f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and $f(0) = 0$, then $\lim_{x \to 0} \frac{1}{x^2}\int_0^x f(t)\,dt$
[31 Aug 2021 Shift 1]

Options:

A. equals 0

B. equals 1

C. does not exist

D. equals $\frac{1}{2}$

Answer: D

Solution:

Solution:
$\int_0^x [1 - (f'(t))^2]^{\frac{1}{2}} = \int_0^x f(t)$
Differentiating on both sides,
$\sqrt{1 - [f'(x)]^2} = [f(x)]$
$\Rightarrow 1 - [f'(x)]^2 = [f(x)]^2$
$\Rightarrow 1 - [f(x)]^2 = [f'(x)]^2$
$\Rightarrow f'(x) = \sqrt{1 - [f(x)]^2}$
$\Rightarrow \int \frac{f'(x)dx}{\sqrt{1 - [f(x)]^2}} = \int 1\,dx$
$\Rightarrow \sin^{-1} f(x) = x + C$
$\because f(0) = 0$
$C = 0$
$f(x) = \sin x$
$\Rightarrow \lim_{x \to 0} \frac{\int_0^x f(t)dt}{x^2} = \lim_{x \to 0} \frac{\int_0^x \sin x\,dt}{x^2}$ [applying L'Hopital Rule]

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Question116

If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, y(0) = 1, then y(1) is equal to
[31 Aug 2021 Shift 1]

Options:

A. $\log_2(2 + e)$

B. $\log_2(1 + e)$

C. $\log_2(2e)$

D. $\log_2(1 + e^2)$

Answer: B

Solution:

Solution:
$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y} = \frac{2^x(2^y - 1)}{2^y}$
$\int \frac{2^y}{2^y - 1} dy = \int 2^x dx$
$\frac{\ln(2^y - 1)}{\ln 2} = \frac{2^x}{\ln 2} + C$
As, $y(0) = 1$
$\Rightarrow 0 = \frac{1}{\log 2} + C$
For y(1), $\ln_2(2^y - 1) = 2^1 - 1$
$\Rightarrow 2^y - 1 = e$
$y = \log_2(e + 1)$

Question117

If $\frac{dy}{dx} = \frac{2^x y + 2^y . 2^x}{2^x + 2^{x+y} \log_e 2}$, y(0) = 0, then for y = 1 , the value of x lies in the interval
[31 Aug 2021 Shift 2]

Options:

A. (1, 2)

B. $\left(\frac{1}{2}, 1\right]$

C. (2, 3)

D. $\left(0, \frac{1}{2}\right]$

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Solution:

Solution:
$\frac{dy}{dx} = \frac{2^x y + 2^y.2^x}{2^x + 2^{x+y}\log_e 2}$, $y(0) = 0$
$\Rightarrow \frac{dy}{dx} = \frac{y + 2^y}{1 + 2^y \log_e 2}$
$\Rightarrow \int \frac{1 + 2^y . \log_e 2 dy}{y + 2^y} = \int dx$
$\Rightarrow \log | y + 2^y | = x + C$
$\because y(0) = 0$
$\Rightarrow C = 0$
$\therefore \ln | y + 2^y | = x$
For $y = 1$
$x = \ln | 1 + 2 | = \ln 3$
$\Rightarrow x \in (1, 2)$

Question118

If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0$, $\phi > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to
[31 Aug 2021 Shift 2]

Options:

A. $4\phi(2)$

B. $4\phi(1)$

C. $2\phi(1)$

D. $\phi(1)$

Answer: B

Solution:

Solution:
Given,

$$y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right] \quad ...(i)$$

Let $t = \frac{y}{x}$
$\Rightarrow y = xt$
$\Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$
$\therefore$ Eq. (i) becomes
$t\left(t + x\frac{dt}{dx}\right) = \left(t^2 + \frac{\phi(t^2)}{\ldots}\right)$

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$\Rightarrow xt\frac{dt}{dx} = \frac{\phi(t^2)}{\phi'(t^2)}$
$\Rightarrow \frac{t\phi'(t^2)}{\phi(t^2)}dt = \frac{dx}{x}$
Integrating both sides
$\int \frac{t\phi'(t^2)}{\phi t^{2)}}dt = \int \frac{dx}{x}$
Let $\phi(t^2) = u$
$\Rightarrow t\phi'(t^2)dt = \frac{du}{2}$
$\therefore \frac{1}{2}\int \frac{du}{u} = \int \frac{dx}{x}$
$\Rightarrow \frac{1}{2}\ln u = \ln x + C$
$\Rightarrow \frac{1}{2}\ln \phi(t^2) = \ln x + C$
$\Rightarrow \frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + C$
If $x = 1$, $y = -1$, then $C = \frac{1}{2}\ln(\phi(1))$
$\therefore \frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + 2\ln(\phi(1))$
If $x = 2$, then $\ln\left(\phi\left(\frac{y^2}{4}\right)\right) = \ln 4 + \ln[\phi(1)]$
Or $\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$

Question119

Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that, $y(0) = 7$. Then $y(\pi)$ is equal to [27 Aug 2021 Shift 1]

Options:

A. $2e^{\pi^2} + 5$

B. $e^{\pi^2} + 5$

C. $3e^{\pi^2} + 5$

D. $7e^{\pi^2} + 5$

Answer: A

Solution:

Solution:
Given, $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$, $y(0) = 7$
$\Rightarrow \frac{dy}{dx} + 2\cos x = 2(y + 2\sin x - 5)x$ ...(i)
Let $y + 2\sin x - 5 = t$
$\Rightarrow \frac{dy}{dx} + 2\cos x = \frac{dt}{dx}$
Then, Eq. (i) becomes

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$\Rightarrow \frac{dt}{t} = 2x\,dx$
On integrating
$\ln t = x^2 + C$
$\Rightarrow \ln(y + 2\sin x - 5) = x^2 + C$ ...(ii)
$\therefore y(0) = 7$
$\Rightarrow \ln(7 + 0 - 5) = 0 + C$
$\Rightarrow C = \ln 2$
$\therefore$ From Eq. (ii),
$\ln(y + 2\sin x - 5) = x^2 + \ln 2$
Now, at $x = \pi$
$\ln(y(\pi) + 2\sin\pi - 5) = \pi^2 + \ln 2$
$\Rightarrow \ln(y(\pi) - 5) = \pi^2 + \ln 2$
$\Rightarrow y(\pi) - 5 = e^{\pi^2 + \ln 2}$
$\Rightarrow y(\pi) = 2e^{\pi^2} + 5$

Question120

Let y(x) be the solution of the differential equation $2x^2 dy + (e^y - 2x)dx = 0, x > 0$. If $y(e) = 1$, then $y(1)$ is equal to [26 Aug 2021 Shift 2]

Options:

A. 0

B. 2

C. $\log_e 2$

D. $\log_e(2e)$

Answer: C

Solution:

Solution:
We have, $2x^2 dy + (e^y - 2x)dx = 0$
$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0$
$\frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$
$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2}$ ...(i)
$e^{-y} = t$ ...(ii)
$-e^{-y}dy = dt$
$dy = -\left(\frac{dt}{t}\right)$...(iii)
$\frac{-dt}{dx} - \frac{t}{x} = -\frac{1}{2x^2}$ [From Eq. (i)]
$x\,dt + t\,dx = \frac{dx}{2x}$
$\int d(xt) = \int \frac{dx}{2x}$
$xt = \frac{1}{2}\log(x) + \frac{C}{2}$
$2xe^{-y} = \log x + c$
When $x = e, y = 1$
$2e \cdot e^{-1} = \log e + c$

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$\therefore 2xe^{-y} = \log x + 1$
When x = 1,
$e^{-y} = 0 + 1$
$e^{y} = 2$
$\Rightarrow y = \log_e 2$

Question121

Let us consider a curve, y = f (x) passing through the point (−2, 2) and the slope of the tangent to the curve at any point (x, f (x)) is given by $f(x) + xf'(x) = x^2$ Then
[27 Aug 2021 Shift 1]

Options:

A. $x^2 + 2xf(x) - 12 = 0$

B. $x^3 + 2xf(x) + 12 = 0$

C. $x^3 - 3xf(x) - 4 = 0$

D. $x^2 + 2xf(x) + 4 = 0$

Answer: C

Solution:

Solution:
Given, $f(x) + xf'(x) = x^2$
$\Rightarrow f'(x) + \frac{f(x)}{x} = x$
$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = x \left[\because y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)\right]$
This is linear differential equation.
$\therefore$ Integrating factor IF $= e^{\int \frac{1}{x}dx} = x$
Solution, $y.x = \int x.xdx + C$
or$xy = \frac{x^3}{3} + C$
$\because$ It passes through (−2, 2).
$\therefore -2.2 = \frac{(-2)^3}{3} + C$
$C = -\frac{4}{3}$
Hence, $xf(x) = \frac{x^3}{3} - \frac{4}{3}$
or $x^3 - 3f(x).x - 4 = 0$

Question122

If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation
[27 Aug 2021 Shift 2]

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Options:

A. $y^5 - 2y - 2 = 0$

B. $2y^5 - 2y - 1 = 0$

C. $2y^5 - y^2 - 2 = 0$

D. $y^5 - y^2 - 1 = 0$

Answer: D

Solution:

Solution:
Given, differential equation
$(2x - 10y^3)dy + ydx = 0$
$\Rightarrow \frac{dx}{dy} + \frac{2x}{y} = 10y^2$ ...(i)
This is Linear differential equation
Integrating factor IF $= e^{2\int \frac{1}{y}dy} = y^2$
Solution of differential Eq. (i),
$x.y^2 = \int 10y^2 . y^2 dy + C$
$\Rightarrow xy^2 = 2y^5 + C$ ...(ii)
Solution Eq. (ii) passes through (0, 1)
$\Rightarrow 0.1^2 = 21^5 + C$
$\Rightarrow C = -2$
$\therefore$ Solution of Eq. (i) is
$xy^2 = 2y^5 - 2$
Now, this equation passes through $(2, \beta)$.
$\therefore 2.\beta^2 = 2\beta^5 - 2$
$\Rightarrow \beta^5 - \beta^2 - 1 = 0$
$\Rightarrow \beta$ is root of the equation $y^5 - y^2 - 1 = 0$

Question123

Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x dx + \tan x dy + ydx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \to 0^+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is
[26 Aug 2021 Shift 1]

Options:

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4} - 1$

C. $\frac{\pi}{4} + 1$

D. $\frac{\pi}{4}$

Answer: D

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Solution:

Solution:

We have, $(y+1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$

$\Rightarrow [(y+1)\tan^2 x + y]dx + \tan x\,dy = 0$

$\Rightarrow \frac{dy}{dx} + (y+1)\tan x + \frac{y}{\tan x} = 0$

$\Rightarrow \frac{dy}{dx} + \frac{y\tan^2 x + \tan^2 x + y}{\tan x} = 0$

$\Rightarrow \frac{dy}{dx} + \frac{y\sec^2 x}{\tan x} + \tan^2 x = 0$

$\Rightarrow \frac{dy}{dx} + \left(\frac{\sec^2 x}{\tan x}\right)y = -\tan x$

This is a linear differential equation

$\therefore \text{IF} = e^{\int \frac{\sec^2 x}{\tan x}dx} = e^{\ln(\tan x)} = \tan x$

So, solution is given by

$(y\tan x) = \int -\tan^2 x\,dx = \int (1-\sec^2 x)dx = x - \tan x + C$

$y = x\cot x - 1 + C\cot x$

Now, $\lim_{x\to 0^+} x \,.\, y = 1$

$\Rightarrow \lim_{x\to 0^+} (x^2\cot x - x + Cx\cot x) = 1$

$\Rightarrow \lim_{x\to 0} + \left(x \,.\, \frac{x}{\tan x} - x + \frac{Cx}{\tan x}\right) = 1$

$\Rightarrow 0 - 0 + C = 1$

$\Rightarrow C = 1$

$\therefore y = x\cot x - 1 + \cot x$

Now, $x = \frac{\pi}{4}$

$y = \frac{\pi}{4} - 1 + 1 = \frac{\pi}{4}$

Question124

Let a, b and c be three vectors mutually perpendicular to each other and have same magnitude. If a vector r satisfies.
a × {(r - b) × a} + b × {(r - c) × b} + c × {(r - a) × c} = 0, then r is equal to
[31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

B. $\frac{1}{3}(2\mathbf{a} + \mathbf{b} - \mathbf{c})$

C. $\frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

D. $\frac{1}{2}(\mathbf{a} + \mathbf{b} + 2\mathbf{c})$

Answer: C

Solution:

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$\mathbf{a} \times [(\mathbf{r} - \mathbf{b}) \times \mathbf{a}] + \mathbf{b} \times [(\mathbf{r} - \mathbf{c}) \times \mathbf{b}] + \mathbf{c} \times [(\mathbf{r} - \mathbf{a}) \times \mathbf{c}] = 0$
$\Rightarrow \mathbf{a} \cdot \mathbf{a} (\mathbf{r} - \mathbf{b}) - (\mathbf{a} \cdot (\mathbf{r} - \mathbf{b})) \mathbf{a} + \mathbf{b} \cdot \mathbf{b} (\mathbf{r} - \mathbf{c}) - (\mathbf{b} \cdot (\mathbf{r} - \mathbf{c})) \mathbf{b} + \mathbf{c} \cdot \mathbf{c} (\mathbf{r} - \mathbf{a}) - (\mathbf{c} \cdot (\mathbf{r} - \mathbf{a})) \mathbf{c} = 0$
$\Rightarrow |a|^2 (r - b) - (r \cdot a)a + |b|^2 (r - c) - (r \cdot b)b + |c|^2 (r - a) - (r \cdot c)c = 0$ [$\because$ **a b, c** are mutually perpendicular; $\therefore a \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$]
$\Rightarrow |a|^2 [3r - (a + b + c)] - [(r \cdot a)a + (r \cdot b)b + (r \cdot c)c] = 0$ [$\because |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}|$]
$\Rightarrow |a|^2 [3r - (a + b + c) - r] = 0$
$\therefore 3\mathbf{r} - (\mathbf{a} + \mathbf{b} + \mathbf{c}) - \mathbf{r} = 0$
$\Rightarrow r = \frac{a + b + c}{2}$

Question125

If $y = y(x)$ is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right) dx = 0$; $x > 0$ and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to

[1 Sep 2021 Shift 2]

Options:

A. $\frac{3}{2} - \frac{1}{\sqrt{e}}$

B. $3 + \frac{1}{\sqrt{e}}$

C. $3 + e$

D. $3 - e$

Answer: D

Solution:

Solution:

$x^2 dy + \left(y - \frac{1}{x}\right) dx = 0$

$\Rightarrow x^2 dy + y dx = \frac{dx}{x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

This in linear differential equations

$IF = e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$

$\Rightarrow y \cdot e^{-1/x} = \int e^{-1/x} \cdot \frac{1}{x^3} dx + c$

Let $\frac{-1}{x} = t$

$\Rightarrow \frac{1}{x^2} dx = dt$

$\Rightarrow y \cdot e^{-1/x} = \int -te^t dt + c = -(te^t - e^t) + c$

$\Rightarrow ye^{-1/x} = \frac{1}{x} e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c$

Put $x = 1$, $y = 1$

(1) $e^{-1} = \frac{e^{-1}}{1} + e^{-1} + c$

$\Rightarrow c = -e^{-1}$

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$\Rightarrow y = \frac{1}{x} + 1 - \frac{e^{1/x}}{e}$

Put $x = \frac{1}{2}$

$y = 2 + 1 - \frac{e^2}{e}$

$\Rightarrow y = 3 - e$

Question126

If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation, $(x+1)dy = ((x+1)^2 + y - 3)dx$, $y(2) = 0$ then $y(3)$ is equal to _______.

[NA Jan. 09,2020 (I)]

Answer: 3

Solution:

Solution:

$(x+1)dy = ((x+1)^2 + (y-3))\,dx = 0$

$\Rightarrow \frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$

$\frac{dy}{dx} - \frac{1}{(1+x)}y = (1+x) - \frac{3}{(1+x)}$

I.F. $= e^{-\int \frac{1}{(1+x)}dx} = \frac{1}{(1+x)}$

$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right) = 1 - \frac{3}{(1+x)^2}$

$y = (1+x)\left[x + \frac{3}{(1+x)} + C\right]$

$\because$ At $x = 2$, $y = 0$

$\therefore 0 = 3(2 + 1 + C) \Rightarrow C = -3$

Then, $y = (1+x)\left[x + \frac{3}{1+x} - 3\right]$

Now, at $x = 3$, $y = (1+3)\left[3 + \frac{3}{1+3} - 3\right] = 3$

Question127

Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is:

[Jan. 7,2020 (II)]

Options:

A. $2 - e$

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C. 2

D. $2+e$

Answer: A

Solution:

Solution:

The given differential equation is $\frac{dx}{dy}+x=y^2$

Comparing with $\frac{dx}{dy}+Px=Q$, where $P=1, Q=y^2$

Now, I.F. $=e^{\int 1 . dy}=e^y$

$x . e^y=\int (y^2)e^y . dy=y^2 . e^y-\int 2y . e^y . dy$

$=y^2e^y-2(y . e^y-e^y)+C$

$\Rightarrow x . e^y=y^2e^y-2ye^y+2e^y+C$

$\Rightarrow x=y^2-2y+2+C . e^{-y}$ ....(i)

As $y(0)=1$, satisfying the given differential eqn,

$\therefore$ put $x=0, y=1$ in eqn. (i)

$0=1-2+2+\frac{C}{e}$

$C=-e$

$y=0, x=0-0+2+(-e)(e^{-0})$

$x=2-e$

Question128

The differential equation of the family of curves, $x^2=4b(y+b), b\in R$, is: [Jan. 8, 2020 (II)]

Options:

A. $x(y')^2=x+2yy'$

B. $x(y')^2=2yy'-x$

C. $xy''=y'$

D. $x(y')^2=x-2yy'$

Answer: A

Solution:

Solution:

Since, $x^2=4b(y+b)$

$x^2=4by+4b^2$

$2x=4by'$

$\Rightarrow b=\frac{x}{2y'}$

So, differential equation is

$x^2=\frac{2x}{y'} . y+\left(\frac{x}{y'}\right)^2$

$x(y')^2=2yy'+x$

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Question129

If $f\,2(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $f(0) = 0$, then $f(1)$ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. $\frac{\pi + 1}{4}$

B. $\frac{1}{4}$

C. $\frac{\pi - 1}{4}$

D. $\frac{\pi + 2}{4}$

Answer: A

Solution:

Solution:
$f'(x) = \tan^{-1}(\sec x + \tan x)$

$$= \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$$

Integrate both sides, we get

$$\int (f'(x))dx = \int \left(\frac{\pi}{4} + \frac{x}{2}\right)dx$$

$$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + C$$

$\because f(0) = 0$

$$C = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$$

So, $f(1) = \frac{\pi + 1}{4}$

Question130

If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:
[Jan. 9, 2020 (II)]

Options:

A. $\frac{1}{2}\sqrt{3}e$

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C. $\sqrt{2}e$

D. $\sqrt{3}e$

Answer: D

Solution:

Solution:
The given differential equation,
$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$
Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
Then, $v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1 + v^2}$
$\Rightarrow \frac{1 + v^2}{v^3} dv = -\frac{1}{x} dx$
$\Rightarrow \int \left(\frac{1}{v^3} + \frac{1}{v}\right) dv = \int \frac{-1}{x} dx$
$\Rightarrow \frac{-1}{2}\left(\frac{1}{v^2}\right) + \ln v = -\ln x + c$
$\Rightarrow -\frac{x^2}{2y^2} = -\ln y + c \quad \left[\because v = \frac{y}{x}\right]$
When $x = 1, y = 1$, then $-\frac{1}{2} = c$
$\Rightarrow x^2 = y^2(1 + 2\ln y)$
At $y = e$, $x^2 = e^2(3)$
$\Rightarrow x = \pm\sqrt{3}e$
So, $x = \sqrt{3}e$

Question131

Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1, |x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$ and $y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to:
[Jan. 8, 2020 (I)]

Options:

A. $\frac{2\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

Solution:

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$2y = \sin^{-1} f(x) + C \ = \sin^{-1}(\sin(2\tan^{-1}x)) + C$

$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$

$\frac{\pi}{3} = \frac{\pi}{3} + C \quad \therefore C = 0$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0$

$\Rightarrow 2y = \frac{-\pi}{3} \Rightarrow y = \frac{-\pi}{6}$

Question132

Let $y = y(x)$ be a solution of the differential equation, $\sqrt{1 - x^2}\frac{dy}{dx} + \sqrt{1 - y^2} = 0$, $|x| < 1$

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to:

[Jan. 8, 2020 (I)]

Options:

A. $\frac{\sqrt{3}}{2}$

B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $-\frac{\sqrt{3}}{2}$

Answer: C

Solution:

Solution:
The given differential eqn. is

$\frac{dy}{\sqrt{1 - y^2}} + \frac{dx}{\sqrt{1 - x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}y = \cos^{-1}x$

Hence, $y\left(-\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

Question133

If $y = y(x)$ is the solution of the differential equation, $e^y = e^x$ such that $y(0) = 0$, then $y(l\,)$ is equal to:
[Jan. 7, 2020 (I)]

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A. $1 + \log_e 2$

B. $2 + \log_e 2$

C. 2e

D. $\log_e 2$

Answer: A

Solution:

Solution:

Let $e^y = t$

$e^y \frac{dy}{dx} = \frac{dt}{dx}$

$\therefore d\,td\,x - t = e^x \left[\because e^y \frac{dy}{dx} - e^y = e^x\right]$

$I.F. = e^{\int -1.dx} = e^{-x}$

$t(e^{-x}) = \int e^x . e^{-x} dx \Rightarrow e^{y-x} = x + c$

Put $x = 0$, $y = 0$, then we get $c = 1$

$e^{y-x} = x + 1$

$y = x + \log_e(x + 1)$

Put $x = 1$ $\quad \therefore y = 1 + \log_e 2$

Question134

The general solution of the differential equation $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$ is:
(where C is a constant of integration)
:[Sep. 06, 2020 (I)]

Options:

A. $\sqrt{1 + y^2} + \sqrt{1 + x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1 + x^2} + 1}{\sqrt{1 + x^2} - 1}\right) + C$

B. $\sqrt{1 + y^2} - \sqrt{1 + x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1 + x^2} + 1}{\sqrt{1 + x^2} - 1}\right) + C$

C. $\sqrt{1 + y^2} + \sqrt{1 + x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1 + x^2} - 1}{\sqrt{1 + x^2} + 1}\right) + C$

D. $\sqrt{1 + y^2} - \sqrt{1 + x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1 + x^2} - 1}{\sqrt{1 + x^2} + 1}\right) + C$

Answer: A

Solution:

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$\sqrt{1+x^2}\,.\,\sqrt{1+y^2} = -xy\frac{dy}{dx}$

$\int \frac{\sqrt{1+x^2}}{x}dx = -\int \frac{y}{\sqrt{1+y^2}}dy$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$\Rightarrow \int \frac{\sec^3\theta d\theta}{\tan\theta} = -\int \frac{2y}{2\sqrt{1+y^2}}dy$

$\Rightarrow \int \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\,.\,\cos^2\theta}d\theta = -\sqrt{1+y^2}$

$\Rightarrow \int (\tan\theta\,.\,\sec\theta + \operatorname{cosec}\theta)d\theta = -\sqrt{1+y^2}$

$\Rightarrow \sec\theta + \log_e|\operatorname{cosec}\theta - \cot\theta| = -\sqrt{1+y^2} + C$

$\therefore \sqrt{1+x^2} + \log_e\left|\frac{\sqrt{1+x^2}-1}{x}\right| = -\sqrt{1+y^2} + C$

$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

Question135

If $y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec} x$ is the solution of the differentialequation, $dydx + p(x)y = \frac{2}{\pi}\operatorname{cosec} x,\ 0 < x < \frac{\pi}{2}$, then the function p(x) is equal to: [Sep. 06, 2020 (II)]

Options:

A. $\cot x$

B. $\operatorname{cosec} x$

C. $\sec x$

D. $\tan x$

Answer: A

Solution:

Solution:

$\because y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec} x$

$\frac{dy}{dx} = \frac{2}{\pi}\operatorname{cosec} x - \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec} x\,.\,\cot x$

$= \operatorname{cosec} x\left[\frac{2}{\pi} - \left(\frac{2}{\pi}x - 1\right)\cot x\right]$

$\Rightarrow \frac{dy}{dx} - \frac{2}{\pi}\operatorname{cosec} x = y\cot x$ ....(i)

It is given that,

$\Rightarrow \frac{dy}{dx} - \frac{2}{\pi}\operatorname{cosec} x = -yp(x)$ ....(ii)

By comparision of (i) and (ii), we get

$p(x) = \cot x$

Question136

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If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is :
[Sep. 05, 2020 (I)]

Options:

A. 1

B. -1

C. 0

D. 2

Answer: B

Solution:

Solution:

$\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} = -e^x$

$\int \frac{dy}{2+y} = -\int \frac{e^x}{5+e^x} dx$

$\Rightarrow \log_e |2+y| . \log_e |5+e^x| = \log_e C$

$\Rightarrow |(2+y)(5+e^x)| = C \because y(0) = 1$

$C = 18$

$\therefore (2+y).(5+e^x) = 18$

When $x = \log_e 13$ then $(2+y).18 = 18$

$\Rightarrow 2+y = \pm 1$

$\therefore y = -1, -3$

$\therefore y(\ln 13) = -1$

Question137

The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is (where C is a constant of integration.)
[Sep. 04, 2020 (II)]

Options:

A. $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

B. $x - \log_e(y+3x) = C$

C. $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

D. $x - 2\log_e(y+3x) = C$

Answer: A

Solution:

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Let $y + 3x = t$

$\Rightarrow \frac{dy}{dx} + 3 = \frac{dt}{dx}$

Putting these value in given differential equation

$\frac{dt}{dx} = \frac{t}{\log_e t}$

$\Rightarrow \int \frac{\log_e t}{t} dt = \int dx$

$\Rightarrow \frac{(\log_e t)^2}{2} = x - C$

$\Rightarrow x - \frac{1}{2}(\ln(y + 3x))^2 = C$

Question138

Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$

If $f(x) = 1$, then x is equal to :
[Sep. 04, 2020 (II)]

Options:

A. $\frac{1}{e}$

B. 2e

C. $\frac{1}{2e}$

D. e

Answer: A

Solution:

Solution:

$\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$

$\Rightarrow \lim_{t \to x} \frac{2t f^2(x) - 2x^2 f(t) . f(t)}{1} = 0$

Using L'Hospital's rule

$\Rightarrow f(x) = x f'(x)$

$\int \frac{f'(x)}{f(x)} dx = \int \frac{1}{x} dx$

$\log_e f(x) = \log_e x + \log_e C$

$\Rightarrow f(x) = Cx \quad \because f(1) = e$

$\Rightarrow C = e$; so $f(x) = ex$

When $f(x) = 1 = ex \Rightarrow x = \frac{1}{e}$

Question139

The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$,

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[Sep. 03, 2020 (I)]

Options:

A. $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$

B. $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{x}}{2}\right) + 2\right)$

C. $y^2 = 1 + y\log_e\left(\frac{1+e^{x}}{2}\right)$

D. $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

Answer: C

Solution:

Solution:

$\int\left(\frac{y^2+1}{y^2}\right)dy = \int\frac{e^x dx}{e^x+1}$

$\Rightarrow y - \frac{1}{y} = \log_e\left|e^x+1\right| + c$

$\because$ Passes through (0,1)

$\therefore c = -\log_e 2$

$\Rightarrow y^2 - 1 = y\log_e\left(\frac{e^x+1}{2}\right)$

$\Rightarrow y^2 = 1 + y\log_e\left(\frac{e^x+1}{2}\right)$

Question140

If $x^3 dy + xy dx = x^2 dy + 2y dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to : [Sep. 03,2020 (II)]

Options:

A. $\frac{3}{2} + \sqrt{e}$

B. $\frac{3}{2}\sqrt{e}$

C. $\frac{1}{2} + \sqrt{e}$

D. $\frac{\sqrt{e}}{2}$

Answer: B

Solution:

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$\Rightarrow (x^3 - x^2)dy = (2 - x)y dx$

$\Rightarrow \frac{dy}{y} = \frac{2 - x}{x^2(x - 1)} dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{2 - x}{x^2(x - 1)} dx$

Let $\frac{2 - x}{x^2(x - 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 1}$

$\Rightarrow 2 - x = A(x - 1) + B(x - 1) + Cx^2$

Compare the coefficients of x, x^2 and constant term.

C = 1, B = −2 and A = −1

$\therefore \int \frac{dy}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x - 1} \right\} dx$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln\left|x - 1\right| + C$

$\therefore y(2) = e$

$\Rightarrow 1 = -\ln 2 + 1 + 0 + C$ [$\because$ log e = 1]

$\Rightarrow C = \ln 2$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln\left|x - 1\right| + \ln 2$

At x = 4,

$\Rightarrow \ln y(4) = -\ln 4 \frac{1}{2} + \ln 3 + \ln 2$

$\Rightarrow \ln y(4) = \ln\left(\frac{3}{2}\right) + \frac{1}{2} = \ln\left(\frac{3}{2} e^{1/2}\right)$ [$\because \log m + \log n = \log(mn)$]

$\Rightarrow y(4) = \frac{3}{2} e^{1/2}$

Question141

Let y = y(x) be the solution of the differential equation,$2 + \sin xy + 1 . \frac{dy}{dx}$ $= -\cos x$, y > 0, y(0) = 1. If y(π) = a and $\frac{dy}{dx}$ at x = π is b, then the ordered pair (a, b) is equal to :
[Sep. 02, 2020 (I)]

Options:

A. $\left(2, \frac{3}{2}\right)$

B. (1,-1)

C. (1,1)

D. (2,1)

Answer: C

Solution:

Solution:

The given differential equation is

$\frac{2 + \sin x}{y + 1} \frac{dy}{dx} = -\cos x, y > 0$

$\Rightarrow \frac{dy}{y + 1} = -\frac{\cos x}{2 + \sin x} dx$

Integrate both sides,

$\int \frac{dy}{y + 1} = \int \frac{(-\cos x) dx}{2 + \sin x}$

$\ln|y + 1| = -\ln|2 + \sin x| + \ln C$

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$\because y(0) = 1 \Rightarrow \ln 4 = \ln C \Rightarrow C = 4$
$\therefore (y+1)(2+\sin x) = 4$
$\Rightarrow y = \frac{4}{2+\sin x} - 1$
$\therefore y = \frac{2-\sin x}{2+\sin x} \Rightarrow y(\pi) = \frac{2-\sin \pi}{2+\sin \pi} = 1$
$\Rightarrow a = 1$
Now, $\frac{dy}{dx} = \frac{(2+\sin x)(-\cos x) - (2-\sin x) . \cos x}{(2+\sin x)^2}$
$\left.\frac{dy}{dx}\right|_{x=\pi} = 1 \Rightarrow b = 1$
Ordered pair (a, b) = (1, 1)

Question142

If a curve $y = f(x)$, passing through the point (1,2) , is the solution of the differential equation, $2x^2 dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to [Sep. 02, 2020 (II)]

Options:

A. $\frac{1}{1+\log_e 2}$

B. $\frac{1}{1-\log_e 2}$

C. $1+\log_e 2$

D. $\frac{-1}{1+\log_e 2}$

Answer: A

Solution:

Solution:
$\frac{dy}{dx} = \frac{2xy+y^2}{2x^2}$
It is homogeneous differential equation.
$\therefore$ Put $y = vx$
$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{2} \Rightarrow \int 2\frac{dv}{v^2} = \int \frac{dx}{x}$
$\Rightarrow \frac{-2}{v} = \log_e x + c \Rightarrow \frac{-2x}{y} = \log_e x + c$
Put $x = 1$, $y = 2$, we get $c = -1$
$\Rightarrow \frac{-2x}{y} = \log_e x - 1$
Hence, put $x = \frac{1}{2} \Rightarrow y = \frac{1}{1+\log_e 2}$

Question143

Let $y = y(x)$ be the solution of the differential equation

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If y(π / 3) = 0, then y(π / 4) is equal to :
[Sep. 05, 2020 (II)]

Options:

A. $2-\sqrt{2}$

B. $2+\sqrt{2}$

C. $\sqrt{2}-2$

D. $\frac{1}{\sqrt{2}}-1$

Answer: C

Solution:

Solution:
$\frac{dy}{dx}+2y\tan x=2\sin x$
I.F. $=e^{\int 2\tan x\,dx}=\sec^2 x$
The solution of the differential equation is
$y\times I.F=\int I.F\times 2\sin x\,dx+C$
$\Rightarrow y.\sec^2 x=\int 2\sin x.\sec^2 x\,dx+C$
$\Rightarrow y\sec^2 x=2\sec x+C$ ......(i)
When $x=\frac{\pi}{3}$, $y=0$; then $C=-4$
$\therefore$ From(i), $y\sec^2 x=2\sec x-4$
$\Rightarrow y=\frac{2\sec x-4}{\sec^2 x}\Rightarrow y\left(\frac{\pi}{4}\right)=\sqrt{2}-2$

Question144

Let y = y(x) be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x)$, $x>0$. If $y(\pi)=\pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to :
[Sep. 04, 2020 (I)]

Options:

A. $2+\frac{\pi}{2}$

B. $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

C. $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

D. $1+\frac{\pi}{2}$

Answer: A

Solution:

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Solution:

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)$

I.F. $= e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

$\therefore \int d\left(\frac{y}{x}\right) = \int (x\cos x + \sin x)dx$

$\Rightarrow \frac{y}{x} = x\sin x + C \because y(\pi) = \pi \Rightarrow C = 1$

$y = x^2\sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

$y' = 2x\sin x + x^2\cos x + 1$

$y'' = 2\sin x - x^2\sin x \Rightarrow y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$

$\therefore y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi}{2} = 2 + \frac{\pi}{2}$

Question145

If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to:

[Jan. 09, 2019 (I)]

Options:

A. $\frac{7}{64}$

B. $\frac{1}{4}$

C. $\frac{49}{16}$

D. $\frac{13}{16}$

Answer: C

Solution:

Solution:

Since, $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x$

$I.F. = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Solution of differential equation is:

$y.x^2 = \int x.x^2 dx$

$y.x^2 = \frac{x^4}{4} + C$ ....(1)

$\because y(1) = 1$

$\therefore C = \frac{3}{4}$

Then, from equation (1)

$y.x^2 = \frac{x^4}{4} + \frac{3}{4}$

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$\therefore y\left(\frac{1}{2}\right) = \frac{1}{16} + 3 = \frac{49}{16}$

Question146

Let $f : [0, 1] \to R$ be such that $f(xy) = f(x), f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is[Jan. 09,2019 equal to:
[Jan. 09, 2019 (II)]

Options:

A. 3

B. 4

C. 2

D. 5

Answer: A

Solution:

Solution:
$f(xy) = f(x)f(y)$ ......(1)
Put $x = y = 0$ in (1) to get $f(0) = 1$
Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$
$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$
imply $f(0) = 0$
Hence, $f(0) = 1$ and $f(1) = 1$
By first principle derivative formula,
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\lim_{h \to 0} f(x) \left(\frac{f\left(1 + \frac{h}{x}\right) - f(1)}{h} \right)$
$\Rightarrow f'(x) = \frac{f(x)}{x} f'(1)$
$\Rightarrow \frac{f'(x)}{f(x)} = \frac{k}{x} \Rightarrow \ln f(x) = k \ln x + c$
$f(1) = 1 \Rightarrow \ln 1 = k \ln 1 + c \Rightarrow c = 0$
$\Rightarrow \ln f(x) = k \ln x \Rightarrow f(x) = x^k$ but $f(0) = 1$
$\Rightarrow k = 0$
$\therefore f(x) = 1$
$\frac{dy}{dx} = f(x) = 1 \Rightarrow y = x + c,\ y(0) = 1 \Rightarrow c = 1$
$\Rightarrow y = x + 1$
$\therefore y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$

Question147

If $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals:

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[10 Jan 2019 I]

Options:

A. $\frac{1}{3}+e^{6}$

B. $\frac{1}{3}$

C. $-\frac{4}{3}$

D. $\frac{1}{3}+e^{3}$

Answer: A

Solution:

Solution:

Given, $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}$

$\frac{dy}{dx}=\sec^2 x(1-3y)$

$\Rightarrow \int\frac{dy}{(1-3y)}=\int\sec^2 x dx$

$\Rightarrow -\frac{1}{3}\ln\left|1-3y\right| = \tan x + C$ ...... (i)

$\because y\left(\frac{\pi}{4}\right)=\frac{4}{3}$(Given)

$\Rightarrow -\frac{1}{3}\ln\left|1-4\right| = \tan\frac{\pi}{4}+C$

$\Rightarrow -\frac{1}{3}\ln 3 = C+1 \Rightarrow C=-1-\frac{1}{3}\ln 3$

$\therefore$ in eq. (i), we get

$\frac{-1}{3}\ln\left|1-3y\right| = \tan x-1-\frac{1}{3}\ln 3$

Put, $x=-\frac{\pi}{4}$

$\Rightarrow -\frac{1}{3}\ln\left|1-3y\right| = \tan\left(-\frac{\pi}{4}\right)-1-\frac{1}{3}\ln 3$

$=-1-1-\frac{1}{3}\ln 3$

$\Rightarrow \ln|1-3y| = 6+\ln 3$

$\Rightarrow \ln|13-y| = 6 \Rightarrow \left|\frac{1}{3}-y\right| = e^{6} \Rightarrow y=\frac{1}{3}\pm e^{6}$

Question148

The curve amongst the family of curves represented by the differential equation, $(x^2-y^2)dx+2xydy=0$ which passes through (1, 1), is: [Jan. 10, 2019 (II)]

Options:

A. a circle with centre on the x-axis.

B. an ellipse with maor axis along the y-axis.

C. a circle with centre on the y-axis.

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D. a hyperbola with transverse axis along the x-axis

Answer: A

Solution:

Solution:
$(x^2-y^2)dx+2xydy=0$
$y^2dx-2xydy=x^2dx$
$2xydy-y^2dx=-x^2dx$
$d(xy^2)=-x^2dx$
$\frac{xd(y^2)-y^2d(x)}{x^2}=-dx$
$d\left(\frac{y^2}{x}\right)=-dx$
$\int d\left(\frac{y^2}{x}\right)=-\int dx$
$\frac{y^2}{x}=-x+C$
Since, the above curve passes through the point (1,1)
Then, $\frac{1^2}{1}=-1+C\Rightarrow C=2$
Now, the curve (1) becomes
$y^2=-x^2+2x$
$\Rightarrow y^2=-(x-1)^2+1$
$(x-1)^2+y^2=1$
The above equation represents a circle with centre (1,0) and centre lies on x -axis.

Question149

Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x>0)$ and $f(1)\neq 4$. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$

[Jan. 10, 2019 (II)]

Options:

A. exists and equals $\frac{4}{7}$.

B. exists and equals 4 .

C. does not exist.

D. exists and equals 0 .

Answer: B

Solution:

Solution:
Let $y=f(x)$
$\frac{dy}{dx}+\left(\frac{3}{4x}\right)y=7$
I.F. $=e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln x}=x^{\left(\frac{3}{4}\right)}$

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$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx + C$

$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)} + C = 4x^{\frac{7}{4}} + C$

$y = 4x + Cx^{-\frac{3}{4}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$

$\Rightarrow \lim_{x \to 0^-} x \cdot f\left(\frac{1}{x}\right) = \lim_{x \to 0^+} \left(4 + Cx^{\frac{7}{4}}\right) = 4$

Question150

The solution of the differential equation, $\frac{dy}{dx} = (x - y)^2$, when $y(1) = 1$, is: [Jan. 11, 2019(II)]

Options:

A. $\log_e \left|\frac{2 - x}{2 - y}\right| = x - y$

B. $-\log_e \left|\frac{1 - x + y}{1 + x - y}\right| = 2(x - 1)$

C. $-\log_e \left|\frac{1 + x - y}{1 - x + y}\right| = x + y - 2$

D. $\log_e \left|\frac{2 - y}{2 - x}\right| = 2(y - 1)$

Answer: B

Solution:

Solution:
The given differential equation

$\frac{dy}{dx} = (x - y)^2$ ....(1)

Let $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

Now, from equation (1)

$\left(1 - \frac{dt}{dx}\right) = (t)^2$

$\Rightarrow 1 - t^2 = \frac{dt}{dx} \Rightarrow \int dx = \int \frac{dt}{1 - t^2}$

$\Rightarrow -x = \frac{1}{2 \times 1} \ln\left|\frac{t - 1}{t + 1}\right| + c$

$\Rightarrow -x = \frac{1}{2} \ln\left|\frac{x - y - 1}{x - y + 1}\right| + c$

$\because$ The given condition $y(1) = 1$

$-1 = \frac{1}{2} \ln\left|\frac{1 - 1 - 1}{1 - 1 + 1}\right| + c \Rightarrow c = -1$

Hence, $2(x - 1) = -\ln\left|\frac{1 - x + y}{1 - y + x}\right|$

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Question151

If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$, $x>0$, where $y(1)=\frac{1}{2}e^{-2}$, then:
[Jan. 11, 2019 (I)]

Options:

A. $y(\log_e 2)=\log_e 4$

B. $y(\log_e 2)=\frac{\log_e 2}{4}$

C. y(x) is decreasing in $\left(\frac{1}{2},1\right)$

D. y(x) is decreasing in (0,1)

Answer: C

Solution:

Solution:
Given differential equation is,
$\frac{dy}{dx}+\left(2+\frac{1}{x}\right)y=e^{-2x}, x>0$
$IF=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ln x}=xe^{2x}$
Complete solution is given by
$y(x).xe^{2x}=\int xe^{2x}.e^{-2x}dx+c$
$=\int xdx+c$
$y(x).e^{2x}.x=\frac{x^2}{2}+c$
Given, $y(1)=\frac{1}{2}e^{-2}$
$\therefore \frac{1}{2}e^{-2}.e^2.1=\frac{1}{2}+c\Rightarrow c=0$
$\therefore y(x)=\frac{x^2}{2}.\frac{e^{-2x}}{x}$
$y(x)=\frac{x}{2}.e^{-2x}$
Differentiate both sides with respect to x,
$y'(x)=\frac{e^{-2x}}{2}(1-2x)<0\ \forall x\in\left(\frac{1}{2},1\right)$
Hence, y(x) is decreasing in $\left(\frac{1}{2},1\right)$

Question152

Let y = y(x) be the solution of the differential equation,
$x\frac{dy}{dx}+y=x\log_e x$, (x > 1). If $2y(2)=\log_e 4-1$, then y(e) is equal to :
[Jan. 12, 2019 (I)]

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A. $-\frac{e}{2}$

B. $-\frac{e^2}{2}$

C. $\frac{e}{4}$

D. $\frac{e^2}{4}$

Answer: C

Solution:

Solution:
Consider the differential equation,
$\frac{dy}{dx} + \frac{y}{x} = \log_e x$
$\because I F = e^{\int \frac{1}{x} dx} = x$
$\therefore yx = \int x \ln x \, dx$
$\Rightarrow xy = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$
$\Rightarrow xy = \frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + c$
Given, $2y(2) = \log_e 4 - 1$
$\therefore 2y = 2\ln 2 - 1 + c$
$\Rightarrow \ln 4 - 1 = \ln 4 - 1 + c$
i.e. $c = 0$
$\Rightarrow xy = \frac{x^2}{2} \ln x - \frac{x^2}{4}$
$\Rightarrow y = \frac{x}{2} \ln x - \frac{x}{4}$
$\Rightarrow y(e) = \frac{e}{2} - \frac{e}{4} = \frac{e}{4}$

Question153

If a curve passes through the point (1,-2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point :
[Jan. 12, 2019 (II)]

Options:

A. (3,0)

B. $(\sqrt{3}, 0)$

C. (-1,2)

D. $(-\sqrt{2}, 1)$

Answer: B

Solution:

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Solution:

$\because$ Slope of the tangent $= \frac{x^2 - 2y}{x}$

$\because \frac{dy}{dx} = \frac{x^2 - 2y}{x}$

$\frac{dy}{dx} + \frac{2}{x}y = x$

$I.F = e^{\int \frac{2}{x}} = e^{2\ln x} = x^2$

Solution of equation

$y \cdot x^2 = \int x \cdot x^2 dx$

$x^2 y = \frac{x^4}{4} + C$

$\therefore$ curve passes through point (1,-2)

$(1)^2(-2) = \frac{1^4}{4} + C$

$\Rightarrow C = \frac{-9}{4}$

Then, equation of curve

$y = \frac{x^2}{4} - \frac{9}{4x^2}$

Since, above curve satisfies the point.

Hence, the curve passes through $(\sqrt{3}, 0)$

Question154

The general solution of the differential equation $(y^2 - x^3) dx - xy dy = 0 (x \neq 0)$ is: (where c is a constant of integration) [April 12, 2019 (II)]

Options:

A. $y^2 - 2x^2 + cx^3 = 0$

B. $y^2 + 2x^3 + cx^2 = 0$

C. $y^2 + 2x^2 + cx^3 = 0$

D. $y^2 - 2x^3 + cx^2 = 0$

Answer: B

Solution:

Solution:

Given differential equation can be written as,

$y^2 dx - xy dy = x^3 dx$

$\Rightarrow \frac{(y dx - x dy)y}{x^2} = x dx \Rightarrow -y d\left(\frac{y}{x}\right) = x dx$

$\Rightarrow -\frac{y}{x} \cdot d\left(\frac{y}{x}\right) = dx \Rightarrow -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow 2x^3 + cx^2 + y^2 = 0$ [Here, $c = 2c_1$]

Question155

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If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:
[April. 09, 2019 (II)]

Options:

A. $\frac{\pi^2}{2\sqrt{3}}$

B. $-\frac{\pi^2}{2}$

C. $-\frac{\pi^2}{2\sqrt{3}}$

D. $-\frac{\pi^2}{4\sqrt{3}}$

Answer: C

Solution:

Solution:
$\cos x\, dy - (\sin x) y dx = 6x dx$
$\Rightarrow \int d(y \cos x) = \int 6x dx \Rightarrow y \cos x = 3x^2 + C$ .....(1)
Given, $y\left(\frac{\pi}{3}\right) = 0$

Putting $x = \frac{\pi}{3}$ and $y = 0$ in eq. (1), we get

$(10) \times \left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C\frac{-\pi^2}{3}$

So, from(1) $y \cos x = 3x^2 - \frac{\pi^2}{3}$

Now, put $x = \frac{\pi}{6}$ in the above equation,

$y\frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3} \Rightarrow \frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$

Question156

Given that the slope of the tangent to a curve y = y(x) at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is:
[April. 08,2019 (II)]

Options:

A. $x \log_e |y| = 2(x - 1)$

B. $x \log_e |y| = -2(x - 1)$

C. $x^2 \log_e |y| = -2(x - 1)$

D. $x \log_e |y| = x - 1$

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Solution:

Solution:
Given $\frac{dy}{dx} = \frac{2y}{x^2}$
Integrating both sides, $\int \frac{dy}{y} = 2\int \frac{dx}{x^2}$
$\Rightarrow \ln|y| = -\frac{2}{x} + C$
Equation (i) passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, i.e. ,(1, 1)
$\therefore C = 2$
Now, $\ln|y| = -\frac{2}{x} + 2$
$x \ln|y| = -2(1-x) \Rightarrow x \ln|y| = 2(x-1)$

Question157

Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$If value of y is 1 when x = 1, then the value of x for which y = 2, is :
[April 12, 2019 (I)]

Options:

A. $\frac{5}{2} + \frac{1}{\sqrt{e}}$

B. $\frac{3}{2} - \frac{1}{\sqrt{e}}$

C. $\frac{1}{2} + \frac{1}{\sqrt{e}}$

D. $\frac{3}{2} - \sqrt{e}$

Answer: B

Solution:

Solution:
Consider the differential equation,
$y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$
$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$
$I.F. = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$
$\therefore x . e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3} dy + c$
Put $-\frac{1}{y} = u \Rightarrow \frac{1}{y^2} dy = du$
$\Rightarrow x . e^{-\frac{1}{y}} = -\int u e^u du + c = -ue^u + e^u + c$
$\Rightarrow x . e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + c$

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$1 = 2 + ce \Rightarrow c = -\frac{1}{e} \Rightarrow x = \left(1 + \frac{1}{y}\right) - \frac{1}{e}e^{\frac{1}{y}}$

On putting y = 2, we get $x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

Question158

If y = y(x) is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 0then $y\left(-\frac{\pi}{4}\right)$ is equal to:
[April 10, 2019 (I)]

Options:

A. $e - 2$

B. $\frac{1}{2} - e$

C. $2 + \frac{1}{e}$

D. $\frac{1}{e} - 2$

Answer: A

Solution:

Solution:
$\frac{dy}{dx} + y\sec^2 x = \sec^2 \times \tan x$
Given equation is linear differential equation.
$IF = e^{\int \sec^2 x dx} = e^{\tan x}$
$\Rightarrow y . e^{\tan x} = \int e^{\tan x}\sec^2 \times \tan x d x$
Put $\tan x = u = \sec^2 x d x = d u$
$ye^{\tan x} = \int e^u u d u \Rightarrow ye^{\tan x} = ue^u - e^u + c$
$\Rightarrow ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$
$\Rightarrow y = (\tan x - 1) + c . e^{-\tan x}$
$\therefore y(0) = 0$ (given) $\Rightarrow 0 = -1 + c \Rightarrow c = 1$
Hence, solution of differential equation,
$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$

Question159

Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y = 1. Then :
[April 10, 2019 (II)]

Options:

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A. $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

B. $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

C. $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

D. $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

Answer: D

Solution:

Solution:
Given differential equation is,
$\frac{dy}{dx}+y\tan x=2x+x^2\tan x$
Here, $P=\tan x$, $Q=2x+x^2\tan x$
I.F. $=e^{\int \tan x\,dx}=e^{\ln|\sec x|}=|\sec x|$
$\therefore y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx$
$=\int x^2\tan x\sec x\,dx+\int 2x\sec x\,dx\ =x^2\sec x+c$
Given $y(0)=1\Rightarrow c=1$
$\therefore y=x^2+\cos x$
Now put $x=\frac{\pi}{4}$ and $x=\frac{-\pi}{4}$ in equation (i),
$y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$ and $y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$
$\Rightarrow y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=0$
$\frac{dy}{dx}=2x-\sin x$
$\therefore y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}}$ and $y'\left(-\frac{\pi}{4}\right)=-\frac{\pi}{2}+\frac{1}{\sqrt{2}}$
$\Rightarrow y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

Question160

The solution of the differential equation $x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$ with $y(1)=1$, is:
[April 09, 2019 (I)]

Options:

A. $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

B. $y=\frac{x^3}{5}+\frac{1}{5x^2}$

C. $y=\frac{x^2}{4}+\frac{3}{4x^2}$

D. $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

Answer: C

Solution:

Solution:

$\frac{dy}{dx} + \frac{2}{x}y = xy(1) = 1(\text{ given })$

Since, the above differential equation is the linear differential equation, then $I.F = e^{\int \frac{2}{x}dx} = x^2$

Now, the solution of the linear differential equation

$y \times x^2 = \int x^3 dx$

$\Rightarrow yx^2 = \frac{x^4}{4} + C \because y(1) = 1$

$\therefore 1 \times 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$

$\therefore$ Solution becomes.

$y = \frac{x^2}{4} + \frac{3}{4x^2}$

Question161

Let y = y(x) be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that y(0) = 0. If $\sqrt{a}y(1) = \frac{\pi}{32}$, then the value of ′a ' is: quad [April
[April 08, 2019 (I)]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{16}$

Answer: D

Solution:

Solution:

$(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$

$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right)y = \frac{1}{(1 + x^2)^2}$

Since, the above differential equation is a linear differential equation

$\therefore$ I.F. $= \int e^{\int 2x.1 + x^2 dx} = e^{\log(1 + x^2)} = 1 + x^2$

Then, the solution of the differential equation

$\Rightarrow y(1 + x^2) = \int \frac{dx}{1 + x^2} + c$

$\Rightarrow y(1 + x^2) = \tan^{-1}x + c$ ....(1)

If $x = 0$ then $y = 0$ (given)

$\Rightarrow 0 = 0 + c$

$\Rightarrow c = 0$

Then, equation (1) becomes

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Now put x = 1 in above equation, then
$2y = \frac{\pi}{4}$
$\Rightarrow 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}\left[\sqrt{a}y(1) = \frac{\pi}{32}\right]$
$\Rightarrow \sqrt{a} = \frac{1}{4}$
$\Rightarrow a = \frac{1}{16}$

Question162

The differential equation representing the family of ellipse having foci either on the x -axis or on the y-axis centre at the origin and passing through the point (0,3) is:
[Online April 16, 2018]

Options:

A. $xyy' + y^2 - 9 = 0$

B. $x + yy'' = 0$

C. $xyy'' + x(y')^2 - yy' = 0$

D. $xyy' - y^2 + 9 = 0$

Answer: C

Solution:

Solution:
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Since, it passes through (0,3)
$\therefore \frac{0}{a^2} + \frac{9}{b^2} = 1$
$\Rightarrow b^2 = 9$
∴ eq. of ellipse becomes:
$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$
differential w.r.t. x, we get;
$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$
$\Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-9}{a^2}$
Again differentiating w.r.t. x, we get;
$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\frac{dy}{dx} = 0$
$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

Question163

The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$

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[Online April 15, 2018]

Options:

A. a circle of radius two

B. a circle of radius one

C. a hyprbola

D. an ellipse

Answer: B

Solution:

Solution:

$(x^2 - y^2)dx + 2xydy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} \Rightarrow v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$

$\Rightarrow \frac{2vdv}{v^2 + 1} = -\frac{dx}{x}$

After integrating, we get

$\ln|v^2 + 1| = -\ln|x| + \ln c$

$\frac{y^2}{x^2} + 1 = \frac{c}{x}$

As curve passes through the point $(1, 1)$, so $1 + 1 = c$

$\Rightarrow c = 2$

$x^2 + y^2 - 2x = 0$, which is a circle of radius one.

Question164

Let $y - y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx} + y\cos x = 4x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then$y\left(\frac{\pi}{6}\right)$ is equal to: [2018]

Options:

A. $\frac{-8}{9\sqrt{3}}\pi^2$

B. $-\frac{8}{9}\pi^2$

C. $-\frac{4}{9}\pi^2$

D. $\frac{4}{9\sqrt{3}}\pi^2$

Answer: B

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Solution:

Solution:
Consider the given differential equation the
$\sin x\,dy + y\cos x\,dx = 4x\,dx$
$\Rightarrow d(y.\sin x) = 4x\,dx$
Integrate both sides
$\Rightarrow y.\sin x = 2x^2 + C$ ....(1)
$\Rightarrow y(x) = \frac{2x^2}{\sin x} + c$ ....(2)
$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$
$\Rightarrow 0 = \frac{\pi^2}{2} + C$
$\Rightarrow C = -\frac{\pi^2}{2}$
Now, put the value of C in (1)
Then, $y\sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution
$\therefore y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$

Question165

Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$,

where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$

If y(0) = 0, then $y\left(\frac{3}{2}\right)$ is
[Online April 15, 2018]

Options:

A. $\frac{e^2 - 1}{2e^3}$

B. $\frac{e^2 - 1}{e^3}$

C. $\frac{1}{2e}$

D. $\frac{e^2 + 1}{2e^4}$

Answer: A

Solution:

Solution:
When $x \in [0, 1]$, then $\frac{dy}{dx} + 2y = 1 \Rightarrow y = \frac{1}{2} + C_1e^{-2x}$
$\because y(0) = 0 \Rightarrow y(x) = \frac{1}{2} - \frac{1}{2}e^{-2x}$
Here, $y(1) = \frac{1}{2} - \frac{1}{2}e^{-2} = \frac{e^2 - 1}{}$

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When $x \notin [0, 1]$, then $\frac{dy}{dx} + 2y = 0 \Rightarrow y = c_2 e^{-2x}$

$\because y(1) = \frac{e^2 - 1}{2} \Rightarrow \frac{e^2 - 1}{2} = c^2 e^{-2} \Rightarrow C_2 = \frac{e^2 - 1}{2}$

$\therefore y(x) \left(\frac{e^2 - 1}{2}\right) e^{-2x} \Rightarrow y\left(\frac{3}{2}\right) = \frac{e^2 - 1}{2e^3}$

Question166

If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to : [2017]

Options:

A. $\frac{4}{3}$

B. $\frac{1}{3}$

C. $-\frac{2}{3}$

D. $-\frac{1}{3}$

Answer: B

Solution:

Solution:

We have $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$

$\Rightarrow \frac{d}{dx}(2 + \sin x)(y + 1) = 0$

On integrating, we get

$(2 + \sin x)(y + 1) = C$

At $x = 0$, $y = 1$ we have

$(2 + \sin 0)(1 + 1) = C$

$\Rightarrow C = 4$

$\Rightarrow y + 1 = \frac{4}{2 + \sin x}$

$y = \frac{4}{2 + \sin x} - 1$

Now $y\left(\frac{\pi}{2}\right) = \frac{4}{2 + \sin\frac{\pi}{2}} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$

Question167

The curve satisfying the differential equation, $y\,dx - (x + 3y^2)\,dy = 0$ and passing through the point (1, 1), also passes through the point : [Online April 8, 2017]

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A. $\left(\frac{1}{4}, -\frac{1}{2}\right)$

B. $\left(-\frac{1}{3}, \frac{1}{3}\right)$

C. $\left(\frac{1}{3}, -\frac{1}{3}\right)$

D. $\left(\frac{1}{4}, \frac{1}{2}\right)$

Answer: B

Solution:

Solution:

$y\,dx - x\,dy - 3y^2\,dy = 0$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$

if $= e^{-\int \frac{1}{y} dy} = e^{-\ln y} = \frac{1}{y}$

$\therefore$ solution is $\frac{x}{y} = \int 3y \cdot \frac{1}{y} dy$

$\Rightarrow \frac{x}{y} = 3y + c$

which passes through (1,1)

$\therefore 1 = 3 + c \Rightarrow c = -2$

$\therefore$ solution becomes $\Rightarrow x = 3y^2 - 2y$

which also passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$

Question168

If a curve $y = f(x)$ passes through the point (1,-1) and satisfies the differential equation, $y(1 + xy)\,dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to: [2016]

Options:

A. $\frac{2}{5}$

B. $\frac{4}{5}$

C. $-\frac{2}{5}$

D. $-\frac{4}{5}$

Answer: B

Solution:

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$\frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx$

$-\frac{x}{y} = \frac{x^2}{2} + C$ as $y(1) = -1 \Rightarrow C = \frac{1}{2}$

Hence, $y = \frac{-2x}{x^2+1} \Rightarrow f\left(\frac{-1}{2}\right) = \frac{4}{5}$

Question169

If $f(x)$ is a differentiable function in the interval $((0, \infty)$ such that $f(a) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$, then $f\left(\frac{3}{2}\right)$ is equal to:
[Online April 9, 2016]

Options:

A. $\frac{23}{18}$

B. $\frac{13}{6}$

C. $\frac{25}{9}$

D. $\frac{31}{18}$

Answer: D

Solution:

Solution:
Let $L = \lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$
Applying L.H. rule
$L = \lim_{t \to x} \frac{2tf(x) - x^2 f'(t)}{1} = 1$
$2xf(x) - x^2 f'(x) = 1$
solving above differential equation, we get
$f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$
Put $x = \frac{3}{2}$
$f\left(\frac{3}{2}\right) = \frac{2}{3} \times \left(\frac{3}{2}\right)^2 + \frac{1}{3} \times \frac{2}{3} = \frac{3}{2} + \frac{2}{9} = \frac{27+4}{18} = \frac{31}{18}$

Question170

The solution of the differential equaiton $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$, and $y(0) = 1$, is given by :
[Online April 10, 2016]

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A. $y^2 = 1 + \frac{x}{\sec x + \tan x}$

B. $y = 1 + \frac{x}{\sec x + \tan x}$

C. $y = 1 - \frac{x}{\sec x + \tan x}$

D. $y^2 = 1 - \frac{x}{\sec x + \tan x}$

Answer: D

Solution:

Solution:

$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$

$2y\frac{dy}{dx} + y^2 \sec x = \tan x$

Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec x = \tan x$

$If = e^{\int \sec x dx} = e^{In(\sec x + \tan x)} = \sec x + \tan x$

$\frac{dt}{dx}(\sec x + \tan x) + t\sec x(\sec x + \tan x)$

$= \tan x(\sec x + \tan x)$

$\int d(t(\sec x + \tan x)) = \int \tan x(\sec x + \tan x)dx$

$t(\sec x + \tan x) = \sec x + \tan x - x$

$t = 1 - \frac{x}{\sec x + \tan x} \Rightarrow y^2 = 1 - \frac{x}{\sec x + \tan x}$

Question171

The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(a)$ is equal to :
[Online April 11, 2015]

Options:

A. 4

B. 3

C. 1

D. 2

Answer: B

Solution:

Solution:
Given differential equation is

$ydx - (x + 2y^2)dy = 0$

$\Rightarrow ydx - xdy - 2y^2dy = 0$

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$\Rightarrow d\left(\frac{x}{y}\right) = 2d\,y$
Integrate both the side
$\Rightarrow \frac{x}{y} = 2y + c$
using $f(-1) = 1$, we get
$c = 1$
$\Rightarrow \frac{x}{y} = 2y + 1$
put $y = 1$, we get $f(a) = 3$

Question172

If y(x) is the solution of the differential equation$(x + 2)\frac{dy}{dx} = x^2 + 4x - 9,\ x \neq -2$ and $y(0) = 0$, then $y(-4)$is equal to :
[Online April 10, 2015]

Options:

A. 0

B. 2

C. 1

D. -1

Answer: A

Solution:

Solution:
$(x + 2)\frac{dy}{dx} = x^2 + 4x - 9x \neq -2$
$\frac{dy}{dx} = \frac{x^2 + 4x - 9}{x + 2}$
$dy = \frac{x^2 + 4x - 9}{x + 2}dx$
$\int dy = \int \frac{x^2 + 4x - 9}{x + 2}dx$
$y = \int\left(x + 2 - \frac{13}{x + 2}\right)dx$
$y = \int(x + 2)dx - 13\int\frac{1}{x + 2}dx$
$y = \frac{x^2}{2} + 2x - 13\log\left|x + 2\right| + c$
Given that $y = (0) = 0$
$0 = -13\log 2 + c$
$y = \frac{x^2}{2} + 2x - 13\log|x + 2| + 13\log 2$
$y(-4) = 8 - 8 - 13\log 2 + 13\log 2 = 0$

Question173

Let y(x) be the solution of the differential equation

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$(x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then $y(e)$ is equal to:
[2015]

Options:

A. 2

B. 2e

C. e

D. 0

Answer: A

Solution:

Solution:
Given, $\frac{dy}{dx} + \left(\frac{1}{x \log x}\right) y = 2$
$I.F. = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$
$y . \log x = \int 2 \log x \, d \, x + c$
$y \log x = 2[x \log x - x] + c$
Put $x = 1$, $y.0 = -2 + c$
$c = 2$
Put $x = e$
$y \log e = 2e(\log e - 1) + c$
$y(e) = c = 2$

Question174

If the differential equation representing the family of all circles touching x -axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)y$, then g(x) equals:
[Online April 9, 2014]

Options:

A. $\frac{1}{2}x$

B. $2x^2$

C. $2x$

D. $\frac{1}{2}x^2$

Answer: C

Solution:

Solution:
Since family of all circles touching x-axis at the origin

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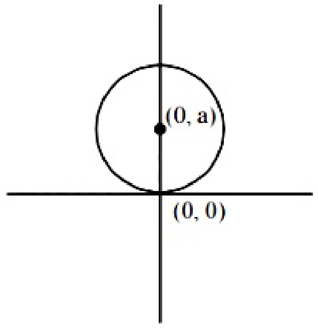

$\therefore$ Eqn is $(x)^2 + (y - a)^2 = a^2$
where (0, a) is the centre of circle.
$\Rightarrow x^2 + y^2 + a^2 - 2ay = a^2$
$\Rightarrow x^2 + y^2 - 2ay = 0$ ....(1)
Differentiate both side w.r.t 'x', we get

$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$

$\Rightarrow x + y\frac{dy}{dx} - a\frac{dy}{dx} = 0$

$\Rightarrow \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}} = a$

Put value of ′a′ in eqn (1), we get

$x^2 + y^2 - 2y\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right] = 0$

$\Rightarrow (x^2 + y^2)\frac{dy}{dx} - 2y^2\frac{dy}{dx} - 2xy = 0$

$\Rightarrow (x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy$

$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy \equiv g(x)y$

Hence, $g(x) = 2x$

Question175

Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$.
If $p(0) = 100$, then p(t) equals:
[2014]

Options:

A. $600 - 500e^{t/2}$

B. $400 - 300e^{-t/2}$

C. $400 - 300e^{t/2}$

D. quad $300 - 200e^{-t/2}$

Answer: C

Solution:

Solution:
Given differential equation is

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By separating the variable, we get

$d\,p(t) = \left[\frac{1}{2}p(t) - 200\right]d\,t$

$\Rightarrow \frac{d\,p(t)}{\frac{1}{2}p(t) - 200} = d\,t$

Integrate on both the sides,

$\int \frac{d\,(p(t))}{\frac{1}{2}p(t) - 200} = \int d\,t$

Let $\frac{1}{2}p(t) - 200 = s \Rightarrow \frac{d\,p(t)}{2} = d\,s$

So, $\int \frac{d\,p(t)}{\left(\frac{1}{2}p(t) - 200\right)} = \int d\,t$

$\Rightarrow \int \frac{2d\,s}{s} = \int d\,t \Rightarrow 2\log s = t + c$

$\Rightarrow 2\log\left(\frac{p(t)}{2} - 200\right) = t + c$

$\Rightarrow \frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$

Using given condition $p(t) = 400 - 300e^{t/2}$

Question176

If the general solution of the differential equation $y' = \frac{y}{x} + \Phi\left(\frac{x}{y}\right)$, for some function Φ, is given by $y\ln|cx| = x$, where c is an arbitrary constant, then $\Phi(2)$ is equal to: [Online April 11, 2014]

Options:

A. 4

B. $\frac{1}{4}$

C. -4

D. $-\frac{1}{4}$

Answer: D

Solution:

Solution:

Given $\frac{d\,y}{d\,x} = \frac{y}{x} + \phi\left(\frac{y}{x}\right)$ .....(1)

Let $\left(\frac{y}{x}\right) = v$ so that $y = xv$

or $\frac{d\,y}{d\,x} = x\frac{d\,v}{d\,x} + v$ ......(2)

from (1) & (2), $x\frac{d\,v}{d\,x} + v = v + \phi\left(\frac{1}{v}\right)$

or, $\frac{d\,v}{\phi\left(\frac{1}{v}\right)} = \frac{d\,x}{x}$

Integrating both sides, we get

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$\int \frac{dx}{x} = \int \frac{dv}{\phi\left(\frac{1}{v}\right)} \Rightarrow \ln x + c = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$ (where c being constant of integration)

But, given $y = \frac{x}{\ln|cx|}$ is the general solution

so that $\frac{x}{y} = \frac{1}{v} = \ln\left|cx\right| = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$

Differentiating w.r.t v both sides, we get

$\phi\left(\frac{1}{v}\right) = \frac{-1}{v^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$

when $\frac{x}{y} = 2$ i.e. $\phi(2) = -\left(\frac{y}{x}\right)^2 = -\left(\frac{1}{2}\right)^2 = \left(\frac{-1}{4}\right)$

Question177

If $\frac{dy}{dx} + y\tan x = \sin 2x$ and $y(0) = 1$, then $y(\pi)$ is equal to: [Online April 19, 2014]

Options:

A. 1

B. -1

C. -5

D. 5

Answer: C

Solution:

Solution:

$\frac{dy}{dx} + y\tan x = \sin 2x$

$I.F = e^{\int \tan x\,dx} = e^{-\log\cos x} = \sec x$

Required solution is

$y(\sec x) = \int \sin 2x \sec x\,dx + c$

$y(\sec x) = \int \frac{2\sin x\cos x}{\cos x}dx + c$

$y(\sec x) = 2\int \sin x\,dx + c$

$y(\sec x) = -2\cos x + c$ ....(1)

Given $y(0) = 1$

∴ put $x = 0$ and $y = 1$, we get

$1(\sec 0) = -2\cos 0 + c$

$\Rightarrow c = 1 + 2 \Rightarrow c = 3$

∴ from eqn (1), we have

$y\sec x = -2\cos x + 3$

To find $y(\pi)$, put $x = \pi$ in eqn (2), we get

$y(\sec\pi) = -2\cos\pi + 3$

$y = -2(-1)(-1) + 3(-1) = -2 - 3 = -5$

Question178

The general solution of the differential

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[Online April 12, 2014]

Options:

A. $y\sqrt{\tan x} = x + c$

B. $y\sqrt{\cot x} = \tan x + c$

C. $y\sqrt{\tan x} = \cot x + c$

D. $y\sqrt{\cot x} = x + c$

Answer: D

Solution:

Solution:

Given, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

or , $\frac{dy}{dx} = \frac{y}{\sin 2x} + \sqrt{\tan x}$

or, $\frac{dy}{dx} - y\operatorname{cosec} 2x = \sqrt{\tan x}$ ......(1)

Now, integrating factor (I.F) $= e^{\int -\operatorname{cosec} 2x}$

or, $I.F = e^{-\frac{1}{2}\log|\tan x|} = e^{\log(\sqrt{\tan x})^{-1}}$

$= \frac{1}{\sqrt{\tan x}} = \sqrt{\cot x}$

Now, general solution of eq. (1) is written as

$y(I.F) = \int Q(I.F.)dx + c$

$\therefore y\sqrt{\cot x} = \int \sqrt{\tan x}.\sqrt{\cot x}dx + c$

$\therefore y\sqrt{\cot x} = \int 1.dx + c$

$\therefore y\sqrt{\cot x} = x + c$

Question179

Statement-1: The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P.

Statement-2: The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1.

[Online April 9, 2013]

Options:

A. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.

B. Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1.

C. Statement-1 is true; Statement-2 is false.

D. Statement-1 is false; Statement-2 is true.

Answer: B

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Solution:
Statement -1: $y^2 = \pm 4ax$
$\Rightarrow \frac{dy}{dx} = \pm 2a \cdot \frac{1}{y} \Rightarrow \frac{dy}{dx} \propto \frac{1}{y}$
Statement - 2: $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$
Thus both statements are true but statement- 2 is not a correct explanation for statement-1.

Question180

At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is
[2013]

Options:

A. 2500

B. 3000

C. 3500

D. 4500

Answer: C

Solution:

Solution:
Given, Rate of change is $\frac{dP}{dx} = 100 - 12\sqrt{x}$
$\Rightarrow dP = (100 - 12\sqrt{x})dx$
By integrating
$\int dP = \int (100 - 12\sqrt{x})dx$
$P = 100x - 8x^{3/2} + C$
Given when $x = 0$ then $P = 2000$
$\Rightarrow C = 2000$
Now when $x = 25$ then
$P = 100 \times 25 - 8 \times (25)^{\frac{3}{2}} + 2000 \; = 4500 - 1000$
$\Rightarrow P = 3500$

Question181

If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1 - \frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is -2 is :
[Online April 23, 2013]

Options:

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A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $-\frac{5}{2}$

Answer: A

Solution:

Solution:
Slope $= \frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\Rightarrow \int dy = \int \left(1 - \frac{1}{x^2}\right) dx$

$\Rightarrow y = x + \frac{1}{x} + C$, which is the equation of the curve since curve passes through the point $\left(2, \frac{7}{2}\right)$

$\therefore \frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$

$\therefore y = x + \frac{1}{x} + 1$

when $x = -2$, then $y = -2 + \frac{1}{-2} + 1 = \frac{-3}{2}$

Question182

Consider the differential equation :

$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$

Statement-1: The substitution $z = y^2$ transforms the above equation into a first order homogenous differential equation.

Statement-2: The solution of this differential equation is $y^2 e^{-y^2/x} = C$

[Online April 22, 2013]

Options:

A. Both statements are false.

B. Statement- 1 is true and statement- 2 is false.

C. Statement- 1 is false and statement- 2 is true.

D. Both statements are true.

Answer: D

Solution:

Solution:
Given differential equation is

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By substituting $z = y^2$, we get diff. eqn. as

$$\frac{dz}{dx} = \frac{2z^2}{2(xz - x^2)} = \frac{z^2}{xz - x^2}$$

Now, $\frac{dx}{dz} = \frac{x}{z} - \frac{x^2}{z^2} = \frac{x}{z}\left[1 - \frac{x}{z}\right] \approx F\left(\frac{x}{z}\right)$

Hence, statement- 1 is true.

Now, $y^2 e^{-y^2/x} = C$ satisfies the given diff. equation

∴ It is the solution of given diff. equation.

Thus, statement- 2 is also true.

Question183

The equation of the curve passing through the origin and satisfying the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

[Online April 25, 2013]

Options:

A. $(1 + x^2)y = x^3$

B. $3(1 + x^2)y = 2x^3$

C. $(1 + x^2)y = 3x^3$

D. $3(1 + x^2)y = 4x^3$

Answer: D

Solution:

Solution:

Given differential equation is

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right)y = \frac{4x^2}{1 + x^2}$$

This is linear diff. equation

$$I.F = e^{\int \frac{2x}{1 + x^2}dx} = e^{\log(1 + x^2)} = 1 + x^2$$

∴ Solution is

$$y(1 + x^2) = \int \frac{4x^2}{1 + x^2} \times 1 + x^2 + C$$

$$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + C$$

⇒ Required curve is

$3y(1 + x^2) = 4x^3 (\because C = 0)$

Question184

Statement 1: The degrees of the differential equations $\frac{dy}{dx} + y^2 = x$ and $\frac{d^2y}{dx^2} + y = \sin x$ are equal.

Statement 2: The degree of a differential equation, when it is a

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power of the highest order derivative involved in the differential equation, otherwise degree is not defined.
[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

B. Statement 1 is false, Statement 2 is true.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

Answer: D

Solution:

Solution:
Statement - 1
Given differential equations are $\frac{dy}{dx} + y^2 = x$ and $\frac{d^2y}{dx^2} + y = \sin x$
Their degrees are 1 .
Both have equal degree.
Also, Statement -2 is the correct explanation for Statement -1

Question185

The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:
[2012]

Options:

A. $2 \ln 18$

B. $\ln 9$

C. $\frac{1}{2} \ln 18$

D. $\ln 18$

Answer: A

Solution:

Solution:
Given differential equation is
$\frac{dp(t)}{dt} = 0.5p(t) - 450$
$\Rightarrow \frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$

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$\Rightarrow 2\frac{d\,p(t)}{d\,t} = -[900 - p(t)]$

$\Rightarrow 2\frac{d\,p(t)}{900 - p(t)} = -d\,t$

Integrate both the side, we get

$-2\int\frac{d\,p(t)}{900 - p(t)} = \int d\,t$

Let $900 - p(t) = u$

$\Rightarrow -d\,p(t) = d\,u$

$2\int\frac{d\,u}{u} = \int d\,t \Rightarrow 2\ln u = t + c$ ......(i)

$\Rightarrow 2\ln[900 - p(t)] = t + c$

Given $t = 0$, $p(0) = 850$

$2\ln(50) = c$

Putting in (i)

$\Rightarrow 2\left[\ln\left(\frac{900 - p(t)}{50}\right)\right] = t$

$\Rightarrow 900 - p(t) = 50e^{\frac{t}{2}}$

$\Rightarrow p(t) = 900 - 50e^{\frac{t}{2}}$

let $p(t_1) = 0$

$0 = 900 - 50e^{\frac{t_1}{2}} \therefore t_1 = 2\ln 18$

Question186

Let y(x) be a solution of $\frac{(2 + \sin x)}{(1 + y)}\frac{d\,y}{d\,x} = \cos x$. If $y(0) = 2$,then $y\left(\frac{\pi}{2}\right)$ equals [Online May 7, 2012]

Options:

A. $\frac{5}{2}$

B. 2

C. $\frac{7}{2}$

D. 3

Answer: C

Solution:

Solution:

Given differential equation is

$\frac{(2 + \sin x)}{(1 + y)} \cdot \frac{d\,y}{d\,x} = \cos x$

which can be rewritten as

$\frac{d\,y}{1 + y} = \frac{\cos x}{2 + \sin x}d\,x$

Integrate both the sides, we get

$\int\frac{d\,y}{1 + y} = \int\frac{\cos x\,d\,x}{2 + \sin x}$

$\Rightarrow \log(1 + y) = \log(2 + \sin x) + \log C$

$\Rightarrow 1 + y = C(2 + \sin x)$

Given $y(0) = 2$

$\Rightarrow 1 + 2 = C[2 + \sin 0] \Rightarrow C = \frac{3}{2}$

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Now, $y\left(\frac{\pi}{2}\right)$ can be found as

$1+y=\frac{3}{2}\left(2+\sin\frac{\pi}{2}\right) \Rightarrow 1+y=\frac{9}{2}$

$\Rightarrow y=\frac{7}{2}$

Hence, $y\left(\frac{\pi}{2}\right)=\frac{7}{2}$

Question187

The integrating factor of the differential equation $(x^2-1)\frac{dy}{dx}+2xy=x$ is [Online May 26, 2012]

Options:

A. $\frac{1}{x^2-1}$

B. x^2-1

C. $\frac{x^2-1}{x}$

D. $\frac{x}{x^2-1}$

Answer: B

Solution:

Solution:

Given differential equation is $(x^2-1)\frac{dy}{dx}+2xy=x$

$\Rightarrow \frac{dy}{dx}+\frac{2x}{x^2-1}.y=\frac{x}{x^2-1}$

This is in linear form.

Integrating factor $=e^{\int \frac{2x}{x^2-1}dx}=e^{\int \frac{dt}{t}}$ where $t=x^2-1$

$=e^{\log t}=x^2-1$

Hence, required integrating factor $=x^2-1$

Question188

The general solution of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=x^2$ is [Online May 19, 2012]

Options:

A. $y=cx^{-3}-\frac{x^2}{4}$

B. $y=cx^3-\frac{x^2}{4}$

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D. $y = cx^{-2} + \frac{x^3}{5}$

Answer: D

Solution:

Solution:
Given differential equation is
$\frac{dy}{dx} + \frac{2}{x} . y = x^2$
This is of the linear form.
$\therefore P = \frac{2}{x}, Q = x^2$

$I.F = e^{\int \frac{2}{x} dx} = e^{\log x^2} = x^2$
Solution is
$y . x^2 = \int x^2 . x^2 dx + c = \frac{x^5}{5} + c$
$y = \frac{x^3}{5} + cx^{-2}$

Question189

The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by:
[2011RS]

Options:

A. quad $2y - 3x = 0$

B. $y = \frac{6}{x}$

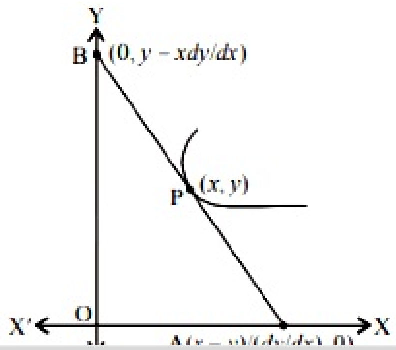
C. $x^2 + y^2 = 13$

D. $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

Answer: B

Solution:

Equation of tangent at P
$Y - y = \frac{dy}{dx}(X - x)$



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X -intercept $= x - \frac{y}{dy/dx}$

Y -intercept $= y - \frac{x dy}{dx}$

Since P is mid-point of A and B

$x - \frac{y}{dy} = 2x$ and $y - \frac{x dy}{dx} = 2y$

$\Rightarrow \frac{-y}{\frac{dy}{dx}} = x$ and $\frac{-x dy}{dx} = y$

$\Rightarrow \frac{dx}{x} = -\frac{dy}{y}$

$\ln y = -\ln x + \ln c$

$y = \frac{c}{x}$

Since the above line passes through the point (2,3) .

$\therefore c = 6$

Hence $y = \frac{6}{x}$ is the required equation.

Question190

Let I be the purchase value of an equipment and V (t) be the value after it has been used for t years. The value V (t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V (T) of the equipment is [2011]

Options:

A. $I - \frac{kT^2}{2}$

B. $I - \frac{k(T-t)^2}{2}$

C. e^{-kT}

D. $T^2 - \frac{1}{k}$

Answer: A

Solution:

Solution:

$\frac{dV(t)}{dt} = -k(T - t)$

$\Rightarrow \int dV(t) = -k \int (T - t) dt$

$V(t) = \frac{k(T-t)^2}{2} + c$

at $t = 0$, $V(t) = I$

$I = \frac{kT^2}{2} + c$

$\Rightarrow c = I - \frac{kT^2}{2}$

$\Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

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Question191

If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to:
[2011]

Options:

A. 5

B. 13

C. -2

D. 7

Answer: D

Solution:

Solution:
$\frac{dy}{dx} = y + 3 \Rightarrow \int \frac{dy}{y+3} = \int dx$
$\Rightarrow \ln|y+3| = x + c$
Given $y(0) = 2$, $\therefore \ln 5 = c$
$\Rightarrow \ln|y+3| = x + \ln 5$
Put $x = \ln 2$, then $\ln|y+3| = \ln 2 + \ln 5$
$\Rightarrow \ln|y+3| = \ln 10$
$\therefore y + 3 = \pm 10 \Rightarrow y = 7, -13$

Question192

Consider the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then x is given by:
[2011RS]

Options:

A. $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$

B. $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

C. $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$

D. $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

Answer: C

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$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$
It is linear differential eqn.
$I.F.=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$
So $x.e^{-\frac{1}{y}}=\int\frac{1}{y^3}e^{-\frac{1}{y}}dy$
Let $\frac{-1}{y}=t$
$\Rightarrow\frac{1}{y^2}dy=dt$
$\Rightarrow I=-\int te^t dt=e^t-te^t=e^{-\frac{1}{y}}+\frac{1}{y}e^{-\frac{1}{y}}+c$
$\Rightarrow xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}+\frac{1}{y}e^{-\frac{1}{y}}+c$
$\Rightarrow x=1+\frac{1}{y}+c.e^{1/y}$
Given y(1) = 1
$\therefore c=-\frac{1}{e}$
$\Rightarrow x=1+\frac{1}{y}-\frac{1}{e}.e^{1/y}$

Question193

Solution of the differential equation $\cos x\,dy=y(\sin x-y)dx$, $0<x<\frac{\pi}{2}$ is [2010]

Options:

A. $y\sec x=\tan x+c$

B. $y\tan x=\sec x+c$

C. $\tan x=(\sec x+c)y$

D. $\sec x=(\tan x+c)y$

Answer: D

Solution:

Solution:
$\cos x\,dy=y(\sin x-y)dx$
$\frac{dy}{dx}=y\tan x-y^2\sec x$
$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$ ....(i)
Let $\frac{1}{y}=t\Rightarrow-\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$
Putting in (i)
$-\frac{dt}{dx}-t\tan x=-\sec x$
$\Rightarrow\frac{dt}{dx}+(\tan x)t=\sec x$
I.F. $=e^{\int\tan x\,dx}=e^{\log|\sec x|}=\sec x$
Solution : $t\sec x=\int\sec x\sec x\,dx$

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Question194

The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1, and c_2 are arbitrary constants, is
[2009]

Options:

A. $y'' = y'y$

B. $yy'' = y'$

C. $yy'' = (y')^2$

D. $y' = y^2$

Answer: C

Solution:

Solution:
We have $y = c_1 e^{c_2 x}$
Differentiate it w.r.t. to x
$\Rightarrow y' = c_1 c_2 e^{c_2 x} = c_2 y$
$\Rightarrow \frac{y'}{y} = c_2$ Differentiate it w.r.t. to x
$\Rightarrow \frac{y''y - (y')^2}{y^2} = 0 \Rightarrow y''y = (y')^2$

Question195

The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is
[2009]

Options:

A. $(x - 2)y'^2 = 25 - (y - 2)^2$

B. $(y - 2)y'^2 = 25 - (y - 2)^2$

C. $(y - 2)^2 y'^2 = 25 - (y - 2)^2$

D. $(x - 2)^2 y'^2 = 25 - (y - 2)^2$

Answer: C

Solution:

Solution:

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$(x-h)^2+(y-2)^2=25$ ......(1)
Differentiating with respect to x, we get
$2(x-h)+2(y-2)\frac{dy}{dx}=0$
$\Rightarrow x-h=-(y-2)\frac{dy}{dx}$
Substituting in equation (1) we get
$(y-2)^2\left(\frac{dy}{dx}\right)^2+(y-2)^2=25$
$\Rightarrow (y-2)^2(y')^2=25-(y-2)^2$

Question196

The solution of the differential equation $\frac{dy}{dy}=\frac{x+y}{x}$ satisfying the condition y(1) = 1 is
[2008]

Options:

A. $y=\ln x+x$

B. $y=x\ln x+x^2$

C. $y=xe^{(x-1)}$

D. $y=x\ln x+x$

Answer: D

Solution:

Solution:
$\frac{dy}{dx}=\frac{x+y}{x}=1+\frac{y}{x}$
It is homogeneous differential eqn.
Putting $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$
we get $v+x\frac{dv}{dx}=1+v\Rightarrow\int\frac{dx}{x}=\int dv$
$\Rightarrow v=\ln x+c\Rightarrow y=x\ln x+cx$
As $y(1)=1$
$\therefore c=1$ So solution is $y=x\ln x+x$

Question197

The differential equation of all circles passing through the origin and having their centres on the x -axis is
[2007]

Options:

A. $y^2=x^2+2xy\frac{dy}{dx}$

B. $y^2=x^2-2xy\frac{dy}{dx}$

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C. $x^2 = y^2 + xy\frac{dy}{dx}$

D. $x^2 = y^2 + 3xy\frac{dy}{dx}$

Answer: A

Solution:

Solution:
General equation of circles passing through origin and having their centres on the x -axis is
$x^2 + y^2 + 2gx = 0$ ......(i)
On differentiating w.r.t x, we get
$2x + 2y \cdot \frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$
Putting in (i)
$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\} \cdot x = 0$
$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx} \cdot y = 0$
$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$

Question198

The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a [2007]

Options:

A. circle

B. hyperbola

C. ellipse

D. parabola.

Answer: B

Solution:

Solution:
Equation of normal at $P(x, y)$ is
$Y - y = -\frac{dx}{dy}(X - x)$
Coordinate of G at X axis is $(X, 0)$ (let)
$\therefore 0 - y = -\frac{dx}{dy}(X - x)$
$\Rightarrow y\frac{dy}{dx} = X - x$
$\Rightarrow X = x + y\frac{dy}{dx}$
$\therefore$ Co-ordinate of G $\left(x + y\frac{dy}{dx}, 0\right)$
Given distance of G from origin = twice of the abscissa of P
$\because$ distance cannot be $-$ve, therefore abscissa x should be +ve

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$\Rightarrow y\,dy = x\,dx$
On Integrating, we have $\frac{y^2}{2} = \frac{x^2}{2} + c_1$
$\Rightarrow x^2 - y^2 = -2c_1$
∴ the curve is a hyperbola

Question199

The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of
[2006]

Options:

A. second order and second degree

B. first order and second degree

C. first order and first degree

D. second order and first degree

Answer: D

Solution:

Solution:
$Ax^2 + By^2 = 1$ ......(i)
Differentiate w.r. to x
$Ax + By\frac{dy}{dx} = 0$ ......(ii)
Again differentiate w.r. to x
$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ ......(iii)
From (ii) and (iii)
$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$
Dividing both sides by $-B$, we get
$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
Therefore order 2 and degree 1 .

Question200

The differential equation representing the family of curves$y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows:
[2005]

Options:

A. order 1, degree 2

B. order 1, degree 1

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C. order 1, degree 3

D. order 2, degree 2

Answer: C

Solution:

Solution:
$y^2 = 2c(x + \sqrt{c})$ .... (i)Differentite it w.r.t. to x
$2yy' = 2c.1$ or $yy' = c$ .... (ii)
[On putting value of c from (ii) in (i)]
$\Rightarrow y^2 = 2yy'(x + \sqrt{yy'})$
On simplifying, we get
$(y - 2xy')^2 = 4yy^3$ .... (iii)
Hence equation (iii) is of order 1 and degree 3 .

Question201

If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is [2005]

Options:

A. $y \log\left(\frac{x}{y}\right) = cx$

B. $x \log\left(\frac{y}{x}\right) = cy$

C. $\log\left(\frac{y}{x}\right) = cx$

D. $\log\left(\frac{x}{y}\right) = cy$

Answer: C

Solution:

Solution:
$\frac{xdy}{dx} = y(\log y - \log x + 1)$
$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$
Put $y = vx$
$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$
$\frac{xdv}{dx} = v\log v \Rightarrow \int \frac{dv}{v\log v} = \int \frac{dx}{x}$
Put $\log v = z$
$\frac{1}{v}dv = dz \Rightarrow \int \frac{dz}{z} = \int \frac{dx}{x}$
$\ln z = \ln x + \ln c$
$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx.$

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Question202

The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is [2004]

Options:

A. $(x^2 + y^2)y' = 2xy$

B. $2(x^2 + y^2)y' = xy$

C. $(x^2 - y^2)y' = 2xy$

D. $2(x^2 - y^2)y' = xy$

Answer: C

Solution:

Solution:
$x^2 + y^2 - 2ay = 0......(1)$
Differentiate w.r.t. to x
$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \Rightarrow a = \frac{x + yy'}{y'}$
Putting in (1) we get, $x^2 + y^2 - 2\left(\frac{x + yy'}{y'}\right)y = 0$
$\Rightarrow (x^2 + y^2)y' - 2xy - 2y^2y' = 0$
$\Rightarrow (x^2 - y^2)y' = 2xy$

Question203

Solution of the differential equation $y\,dx + (x + x^2y)\,dy = 0$ is [2004]

Options:

A. $\log y = Cx$

B. $-\frac{1}{xy} + \log y = C$

C. $\frac{1}{xy} + \log y = C$

D. $-\frac{1}{xy} = C$

Answer: B

Solution:

Solution:
$y\,dx + (x + x^2y)\,dy = 0$
$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2$

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$x^{-2}\frac{dx}{dy}+x^{-1}\left(\frac{1}{y}\right)=-1$
put $x^{-1}=t$, $-x^{-2}\frac{dx}{dy}=\frac{dt}{dy}$ we get,
$-\frac{dt}{dy}+t\left(\frac{1}{y}\right)=-1\Rightarrow\frac{dt}{dy}-\left(\frac{1}{y}\right)t=1$
It is linear differential eqn. in t.
$I.F=e^{\int-\frac{1}{y}dy}=e^{-\log y}=y^{-1}$
∴ Solution is $t(y^{-1})=\int(y^{-1})dy+C$
$\Rightarrow\frac{1}{x}\cdot\frac{1}{y}=\log y+C\Rightarrow\log y-\frac{1}{xy}=C$

Question204

The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively.
[2003]

Options:

A. 2, 3

B. 2, 1

C. 1, 2

D. 3, 2.

Answer: C

Solution:

Solution:
$y^2=4a(x-h)$
Differentiating $2yy_1=4a\Rightarrow yy_1=2a$
Again differentiating, we get $\Rightarrow y_1^{\,2}+yy_2=0$
Degree $=1$, order $=2$

Question205

The solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$, is [2003]

Options:

A. $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

B. $(x-2)=ke^{2\tan^{-1}y}$

C. $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

D. $xe^{\tan^{-1}y}=\tan^{-1}y+k$

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Answer: C

Solution:

Solution:

$(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

$\Rightarrow \frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$

It is form of linear differential equation.

$I.F=e^{\int \frac{1}{(1+y^2)}dy}=e^{\tan^{-1}y}$

$x\left(e^{\tan^{-1}y}\right)=\int \frac{e^{\tan^{-1}y}}{1+y^2}e^{\tan^{-1}y}dy$

$x\left(e^{\tan^{-1}y}\right)=\frac{e^{2\tan^{-1}y}}{2}+C\left[\because \int e^{2x}dx=\frac{e^{2x}}{2}\right]$

$\therefore 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

Question206

The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are
[2002]

Options:

A. $\left(1,\frac{2}{3}\right)$

B. (3,1)

C. (3,3)

D. (1,2)

Answer: C

Solution:

Solution:

$\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3$

$\Rightarrow \left(1+3\frac{dy}{dx}\right)^2=16\left(\frac{d^3y}{dx^3}\right)^3$

Order $=3$, degree 3

Question207

The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$
[2002]

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Options:

A. $\frac{e^{-2x}}{4}$

B. $\frac{e^{-2x}}{4} + cx + d$

C. $\frac{1}{4}e^{-2x} + cx^2 + d$

D. $\frac{1}{4}e^{-4x} + cx + d$

Answer: B

Solution:

$\frac{d^2y}{dx^2} = e^{-2x}$; on integration $\frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$;

Again integrate we get $y = \frac{e^{-2x}}{4} + cx + d$

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